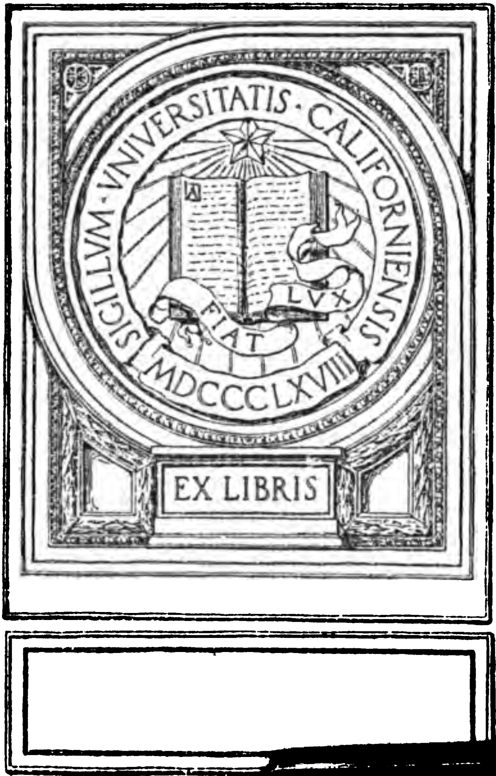
SIGILLVM · VNIVERSITATIS · CALIFORNIENSIS
FIAT LVX
MDCCCLXVIII
EX LIBRIS

THE NEW PHYSICS

By

ALBERT C. CREHORE, PH. D.

ILLUSTRATED

JOURNAL OF ELECTRICITY
SAN FRANCISCO

1920

Lovingly Dedicated
to
MY WIFE

"In Spirit and in Truth"

CONTENTS

of the nucleus of the hydrogen atom depends upon the Rydberg constant. Electro-chemical equivalent gives an accurate value of e/m_H. Coupling this with the previous value of e^2/m_H gives an accurate numerical value of the charge of the electron, e. It also gives an accurate value of the mass of the hydrogen atom, m_H. Combining this value of e with the Bucherer ratio of e to cm_o gives an accurate value of the mass of the electron, m_o.. 22-33

IV

Reasons for making mass have the dimensions of a velocity, $L\,T^{-1}$. First, a new expression has been found for Planck's constant, h, which demands that mass shall have the dimensions of a velocity. Second, the gravitational theory of the author makes the same demand. Meaning of the constant, h. The formula of Einstein experimentally established by Millikan. His experimental value of h from this work is 6.56×10^{-27}. The new theoretical expression for h makes it depend upon the numerical values of the velocity of light and Rydberg's constant only. It is given three different and equivalent forms. The theoretical numerical value of h is 6.558×10^{-27}, very close to Millikan's experimental result. New units of length and time considered, different from the centimeter and the second. Unit of time chosen is the time of one revolution of the electrons in the hydrogen atom. New unit of length is the distance travelled by light during one revolution of the electrons. Velocity of light on this system becomes unity. Twice the Rydberg constant becomes unity. The specific inductive capacity becomes numerically equal to the velocity of light in the C.G.S. system. The units of mass and of energy are large by comparison with the old units. The square of the charge on one electron numerically equal to the mass of the hydrogen atom, but not dimensionally. The mass of the hydrogen atom numerically equal to its energy content, but not dimensionally. The constant h depends upon the specific inductive capacity alone, that is upon the velocity of light alone numerically, but not dimensionally. To omit to express the

V

VI

VII

VIII

PREFACE

This volume has been prepared to fill a demand that seems certain to be created by the publication of my book "The Atom." When the effects of the new expressions for Rydberg's and Planck's constants, connecting them in various ways with the electrical charge and the mass of the hydrogen atom are looked upon with the proper perspective, the appropriateness of the title given to this work, "The New Physics," is more manifest.

The reduction of the two systems of units alone, the electrostatic system and the electromagnetic system, to one common system in terms of length and time introduces us to a new era in Physics. It is difficult to dismiss the thought that we are one step nearer to a satisfactory understanding of the nature of electricity and its intimate connection with the aether of space.

Since this work was prepared the scientific world has been profoundly stirred by the announcement of the results obtained during the total eclipse of the sun of May 29, 1919. For the first time in the history of eclipse observations, beginning with the eclipse of June 8, 1918, efforts have been made to verify the predictions made by Prof. Einstein that rays of light from the stars, passing close to the sun on their way to the earth, would suffer a certain very small deflection. These recent observations seem to agree with the prediction of Dr. Einstein,

and at the least they have focused the attention of the scientific world upon the fundamental basis of the Einstein theory. This theory has sometimes been called "The Relativity Theory of Gravitation," and some reference to it seems to be required in these prefatory remarks in order that this theory and that proposed in these pages may not seem to be conflicting. At first thought it seems as if there is only room for one theory of gravitation, and, if the Einstein theory is receiving confirmation, it may seem to follow that the theory given within these pages is brought into question. It is for the purpose of showing that this is not at all the case that reference to this matter is made here.

An admirable report to The Physical Society of London (1918) has been made by Professor A. S. Eddington on the Relativity Theory of Gravitation, and in summing up the subject on the last page (91) he says, "In this discussion of the law of gravitation, we have not sought, and we have not reached, any ultimate explanation of its cause. A certain connection between the gravitational field and the measurement of space has been postulated, but this throws light rather on the nature of our measurements than on gravitation itself. The relativity theory is indifferent to hypotheses as to the nature of gravitation, just as it is indifferent to hypotheses as to matter and light."

The Einstein theory thus makes no attempt to connect the gravitational force directly with matter or with the motion of the electrons of matter. The

theory given in these pages does make such an attempt, and connects the gravitational force directly with the motion of the electrons of matter, being thus an ultimate explanation of its cause. The two theories are not contradictory in any sense, but rather supplementary. That they are in fact supplementary is still more evident, if we are prepared to admit that the recent eclipse observations have established the theory of relativity, for it is shown within these pages that the theory here described requires for its establishment that the theory of relativity be admitted as established. This is revealed by the necessity for making use of the recent equations developed by Mega Nad Saha and based upon the Minkowski four dimensional space, which involves the theory of relativity.

It is earnestly hoped that these remarks on gravitation will have the desired effect of showing that the Einstein theory deals with but a single phase of the subject, and that it must be subservient to a broad general theory dealing with the cause of the gravitational force.

It is not possible to state the facts that it is desired to present in this volume without the use of some mathematical symbols, but these are restricted to the simplest algebraical equations that the student, who has had a first course in algebra, may read.

ALBERT C. CREHORE.

November, 1919.

ADDED NOTE

The whole work first appeared as a serial in The Journal of Electricity. An Appendix written subsequent to the rest of the work, in April, 1920, on "The Velocity of Propagation of Gravitation" is added because it is thought that there are many who still believe that the gravitational force has a velocity of propagation.

July 4, 1920. A. C. C.
Cleveland, Ohio.

THE NEW PHYSICS

I

In the early study of electricity and magnetism, observations of a great variety of phenomena were made, but at that time they seemed to have no evident connection with one another. When a stick of ebonite is rubbed with flannel the stick becomes electrified and is capable of attracting bits of paper presented to it. When a steel magnet is brought near iron filings they are attracted by the steel, but the bits of paper are not affected by the steel. Neither are the iron filings affected by the electrified rod of ebonite, while the paper is. It cannot be said that electrical science began until some method was discovered by which these varied phenomena could be measured. To do this meant that some kind of law governing the phenomena must first be discovered. We owe to Coulomb (1784) our earliest knowledge of the quantitative laws of electric attraction and repulsion. The result established by the investigations of Coulomb may be thus stated. If two spherical bodies are given fixed charges of electricity

there results an attraction between them when the charges have a different sign, and a repulsion when they have the same sign. Denoting this repulsion by unity, when the distance between the centers of the two spheres is x, say, he proved that the repulsion is 1/4th when the distance is increased to 2x, 1/9th when increased to 3x, etc., without changing the amount of either electrical charge.

And, again, by using metallic spheres, so that the electricity could flow from place to place without obstruction, he found that he could divide the charge that one of the spheres contained into two equal parts simply by bringing into contact with the charged sphere an uncharged sphere of the same radius. The electrical charge of the one then flows over them both, and from the geometrical symmetry it can be seen that when they are separated again each sphere will be equally charged, and each must then contain one-half of the original charge of the first sphere. Dividing the charge in this manner it was observed that the force of repulsion was also divided by two each time without changing at all the distance between the centers of the two spheres. The same thing also applies to the other sphere. When the charge on each was divided by two the force was observed to become one-quarter of its original value. These results may be stated in words as follows. The force which is mutually exerted between two electrified spheres at some distance apart as compared with their radii is directly proportional

to the product of their electrifications and inversely proportional to the square of the distance between their centers. When one quantity is proportional to another, it is equal to the other quantity multiplied by some constant. The law thus established becomes

$$F = C \frac{e_1 e_2}{r^2},$$

where C denotes some constant, e_1 and e_2 the values of the two charges, and r the distance between the centers of the two spheres. But this is not the whole of the law, for it was also found that the force exerted by one charge on the other might be changed in another way without changing either the distance between the centers of the spheres or the amount of charge on either one of them. This could be done simply by inserting between the two spheres a piece of uncharged glass, hard rubber or other material. The effect of this was to reduce the value of the force of repulsion as compared with the force when simply air was present between them. The amount of the reduction of the force depended entirely upon the kind of material inserted between the spheres, and one could tell after a little experience what kind of material it was that was introduced without seeing the material at all, but by observing the change in the value of the force. As a result of this observation a certain number had to be multiplied into the denominator of the equation above given according to the character of the substance in which the two

spheres were placed while conducting the experiment, and this number has been called the specific inductive capacity of the medium because it depends upon the kind of medium only and upon nothing else. It is denoted by the letter k, and the complete law that Coulomb found may be expressed:

$$F = C \frac{e_1 e_2}{k\ r^2},$$

It is found that k has its minimum value in vacuo. It is just a trifle larger for air but is several times larger for glass and gutta percha.

Some such law as this had to be discovered before there was any way by which quantities of electricity could be measured. Assuming that the above is a true relation between the quantities concerned, it is to be observed that there are four kinds of quantities in it,—force, distance, specific inductive capacity, and electrical charge. Only two of these had ever been measured before, force and distance in terms of the centimeter, gram, and the second as units of length, mass and of time. The other two, specific inductive capacity and electrical charge, we seek to obtain from this single equation. But in the nature of things two independent equations are required to obtain the independent values of two unknown quantities. There never has been, however, any other equation than this, and men have been compelled to build up a system of units based upon this one equation, and known as the electrosatic

system of units, by a sort of subterfuge as it were, in which it has been assumed that the specific inductive capacity, k, is numerically equal to unity for a vacuum, and that we are unable to express the unit of specific inductive capacity in terms of length, mass, and time, as force is expressed. This necessarily means that we are also unable to express the other unknown quantity, electrical charge in terms of length, mass and time as force is expressed, because the arbitrary assumption in regard to k must always be reckoned with as unknown.

If, now, it is desired to define the unit of electrical charge by means of the above relation, it may be agreed that there is one unit of electrical charge on each sphere when the distance between the centers of the two spheres, r, is one centimeter, and when the force of repulsion between them is one dyne, the two charges being equal to each other. If the specific inductive capacity, k, is also considered to be unity, this makes all of the quantities in the equation numerically equal to unity except the numeric C. Hence, C must be unity in order that it shall be a true equation. With this definition of the unit, therefore, the constant C may be suppressed, giving simply

$$F = \frac{e_1 e_2}{k\ r^2}.$$

If e_1 and e_2 are equal to each other, this may be written

$$e^2 = k\ F\ r^2.$$

The unit of force, the dyne, is defined as that force

which will produce when continuously applied to one gram of matter a uniform acceleration of it equal to one centimeter per second per second. This definition comes from the known law connecting force with mass and acceleration, namely

$$F = m\ a,$$

where m is the mass in grams, and a the acceleration in centimeters per second per second. By making both m and a unity in this, F bceomes a unit of force. The reason for saying per second per second in referring to acceleration is that the time comes in twice, since acceleration is the rate of change of a velocity. The time comes in once in saying the rate of change, and again in saying velocity because velocity is equal to a distance divided by time. Now, a velocity may be formally expressed by the symbols L/T or $L\,T^{-1}$, where L may denote any length, and T any time. And, an acceleration requires that the velocity be divided again by a time, and so may formally be expressed by the symbols L/T^2 or $L\ T^{-2}$, L denoting any length and T any time as before. According to the last equation above force will be obtained by multiplying this acceleration by a mass, and it may formally be expressed by the symbols

$$F = L\ M\ T^{-2}.$$

No matter what the values of these lengths, masses or times are, as denoted by these symbols, it will always be found that the quantity obtained by multiplying them together in this particular combination will possess all of the qualities of a force.

In a similar fashion every one of the units employed in mechanics, mass, momentum, moment of momentum, force, energy, etc., are capable of being expressed in the symbols L, M and T, and every different kind of unit combines these three symbols in a different way, so that, if we knew what the combination of symbols is, we would also know the nature of the quantity expressed by them. The combinations of symbols above described are ordinarily called the dimensions of the unit or of the quantity, and they are very useful because the combination is fixed for each different kind of unit. When any uncertainty arises as to the kind of quantity which results from a combination of several different kinds of quantities measured by different kinds of units, the matter may easily be decided by writing out the dimensional formula, a procedure which often prevents error.

Let us now return to the expression for the square of electrical charge as given above. The dimensional formula for it is evidently that of a force multiplied by the square of a length and by the unknown quantity, k, giving as a result

$$e^2 = L^3 \, M \, T^{-2} \, k.$$

And, taking the square root, the dimensions of e become

$$e = L^{3/2} \, M^{1/2} \, T^{-1} \, k^{1/2}.$$

These are the dimensions of quantity of electricity which will be found in all of the current tables of dimensions of electrical units in the so-called electro-

static system of units. The presence of the unknown quantity k is required in every one of these units, and it is required because the true nature of k has never been revealed in terms of L, M and T by any other fundamental equation like that established by Coulomb. It may be stated in advance of the proper place that the author has found another fundamental equation, which, combined with this equation of Coulomb, makes it possible to obtain both unknown quantities, e and k, in terms of L, M and T, which thus reveals the true dimensions of e and of k. The result obtained for k is to make it the reciprocal of a velocity, namely $L^{-1} T = T/L$. But this is in advance of the narrative.

At the same time that this law of action between two charges of electricity was found Coulomb also established a similar law of action between two magnetic poles. This may be briefly stated as follows. The force exerted between two magnetic poles at the same distance is directly proportional to the product of the strengths of the poles. And also, the force exerted between two magnetic poles of the same strength but at different distances, is inversely proportional to the squares of the distances. These statements are both included in the formula

$$F = C \frac{m_1 m_2}{r^2},$$

where C is some constant, a numeric, and m_1 and m_2 represent the strengths of the poles, and r the distance between them.

Here again it was found that it was not sufficient to define quantity of magnetism by means of this equation alone, because without changing the distance between the poles or the strengths of them the force exerted by the one upon the other might be altered simply by changing the medium in which the poles were immersed. So an unknown quantity, μ, had to be introduced into the formula. In vacuo the force is a maximum and very nearly the same as in air. So the multiplier, μ, is placed in the equation as was the k in the electrostatic equation, giving

$$F = C\frac{m_1 m_2}{\mu r^2}.$$

There are, as before, four different kinds of quantities concerned in this equation, force, distance, quantity of magnetism, and magnetic permeability, and only two of these have ever been measured or defined before, force and distance. The other two, magnetic permeability and quantity of magnetism, we seek to define by means of this equation. It has in a similar manner been customary to call the magnetic permeability of a vacuum unity and define the unit of quantity of magnetism by taking the distance, r, between the poles unity, and the force unity, whence the constant C must also be unity in order that this may be a true equation, giving,

$$F = \frac{m_1 m_2}{\mu r^2}.$$

If m_1 and m_2 are equal to each other, this may be written

$$m^2 = F \ r^2 \mu.$$

The dimensions of quantity of magnetism are then found from this, as shown, giving

$$m^2 = L^3 M T^{-2} \mu.$$

Except for the change from k to μ these dimensions are precisely the same as those of quantity of electricity on the electrostatic system above given. But, since the dimensions of k and of μ were unknown, there was no means of knowing whether quantity of magnetism and quantity of electricity are the same kinds of quantities or not. In other words the dimensional formulae for electrical and magnetic quantities are robbed of their power of identifying the precise kind of quantity being dealt with until both k and μ are known. We had become so accustomed to relying upon the dimensions of mechanical quantities to determine the kind of quantity without question that it was disconcerting to be compelled to give up this useful tool, and to build up two parallel systems of units, one electrostatic and one electromagnetic unit for every kind of electrical and magnetic quantity. It may be stated in advance of the proper place again that the author has found the dimensions of μ as well as those of k above mentioned, for one of them is automatically determined as soon as the other becomes known, as we shall presently see. The result is that k and μ each have

the same dimensions in terms of length and time, namely the reciprocal of a velocity. According to this, quantity of electrical charge and quantity of magnetism are precisely the same in dimensions, and represent precisely the same kinds of quantities. But this is again in advance of the narrative.

The fundamental conceptions and laws that have led to the establishment of two very different systems of units, the so-called electrostatic system, and the electromagnetic system, with which every student of electricity and magnetism is familiar, because they are now in everyday use, have been outlined above.

II

So long as attention was confined to quantity of electrical charge and to quantity of magnetism alone, it never appeared that there was any need for a unit of quantity of electricity on the electromagnetic system of units, nor of quantity of magnetism on the electrostatic system of units, for these quantities differed so in kind apparently that there was no possible way of measuring one by the unit adopted for the other. It soon appeared, however, that the electric current is a kind of connecting link between these apparently different kinds of quantities, and that electric current could be measured in the units of either system.

This becomes evident when we reflect that two charged conducting spheres may be completely discharged by connecting them with a conducting wire provided their charges are equal and of the opposite sign. In other words the discharge of a condenser, for the two spheres constitute an electrical condenser, causes an electrical current to flow in the discharging wire, and some unit had to be found by which to measure this current. Also some law had to be discovered connecting the value of the current produced by the condenser with the value of the

original charge of it. This law states that the electrical current is equal to the time rate of change of the original charge. It takes time to discharge a condenser, but usually the time is very brief according to our common notions of time, so short indeed that the measurement of the time at first presented some difficulties.

And again, Faraday showed that it was possible to produce a current of electricity without the use of a condenser at all, by the very simple process of moving a magnet in the immediate vicinity of an electrical conductor, and he worked out with consummate skill the exact laws that now bear his name, by which quantity of magnetism and the electric current produced by any definite relative motion between the conductor and the magnetic field become connected by definite expressions.

Now, an electrical current produced by the discharge of a condenser through a wire and again produced by the motion of a magnet in its vicinity, are apparently just the same kind of thing. Indeed the two currents are so much alike that, if the observer of the current itself did not know the source from whence it was derived, he would be unable to tell what the source of it is. It has precisely the same effect upon any of the instruments at his disposal for the purpose of detecting and measuring electrical currents. But, let us now suppose that the observer of the current knows of the source from whence it comes, and applies the known laws in each

case to obtain the measure of precisely the same magnitude of current, the first obtained from the condenser, and the second obtained from the motion of the magnet. He is surprised to find that the numbers obtained when using the electrostatic system of units and the condenser are very large indeed as compared with the numbers obtained when using the magnet, and the electromagnetic system of units. Indeed, a study has been carried out in this way to find the constant ratio between the numbers obtained for the same current on the electrostatic system and on the electromagnetic system of units. This ratio comes out the same every time and is equal to the number approximately 30,000,000,000, which is for brevity always written 3×10^{10}. The larger number is obtained when the electrostatic units are employed, and the smaller when the electromagnetic units are used. The meaning of this is that the electromagnetic unit of electric current is this large number of times greater than the electrostatic unit of current, and that this has resulted simply because of the arbitrary methods of choosing the units of quantity of electricity and of magnetism, not knowing at the time that they implied such different units of current.

Having thus established a unit of current on each system of units, each of the other units in turn receives expression in both systems, including the units of quantity of electricity and of magnetism with which we started. And in a similar manner

the ratios of the magnitudes of each of the units for the same kind of quantity have been measured in each system. The unit of quantity of electricity on the electromagnetic system is found to be 3×10^{10} times the unit of quantity of electricity on the electrostatic system. The unit of electromotive force on the electrostatic system is 3×10^{10} times larger than the unit for electromotive force on the electromagnetic system. The unit of electrical capacity on the electromagnetic system is found to be 9×10^{20} times the unit of capacity on the electrostatic system.

The result thus obtained is very surprising because the ratios between the units always come out the same number, 3×10^{10} or some power of this number. Moreover, this number happens to represent very exactly the numerical value of the velocity with which light travels. This is very strange, and it resulted merely from the manner in which these units were originally defined. It is inevitable to avoid the conclusion that the phenomena of light are in some way very intimately connected with those of electricity and of magnetism. It was James Clerk Maxwell who first pointed out that there is a very definite connection between light, electricity and magnetism in his celebrated treatise on Electricity and Magnetism, in which he developed an electromagnetic theory of light. He predicted in this that, when electromagnetic waves were discovered at some future time, it would be found that they will travel at the same velocity as light waves do. This prediction

has been fulfilled since that time, and the waves now in common use in so-called wireless telegraphy travel at the speed predicted by Maxwell.

One of the reasons for mentioning this subject here is that during the course of developing his electromagnetic theory of light, it was shown to be a necessary condition that the product of the specific inductive capacity and of the magnetic permeability of the medium shall be numerically equal to the reciprocal of the square of the velocity of light. The relation may be thus expressed,

$$k\,\mu = 1/c^2,$$

where c denotes the velocity of light. In other words, the so-called dimensions of the product of k and μ have been known ever since Maxwell's day, namely $L^{-2}T^2$, the reciprocal of the square of a velocity. But this did not give any knowledge of either one of the two quantities independently. It rendered it likely, however, that k and μ depend upon space and time alone rather than upon space, mass and time. This is in agreement with the statement made above that specific inductive capacity may be considered to be the reciprocal of a velocity, for this involves space and time only, and not mass. But, if this is the correct interpretation of specific inductive capacity, the relation found by Maxwell automatically determines the magnetic permeability and makes it also the reciprocal of a velocity, the same kind of a quantity as specific inductive capacity. It is a direct consequence of this that electrical charge and quantity of mag-

netism have the same dimensions and represent the same sort or kind of quantities.

TABLE I.

Quantity		Dimensions on Electrostatic System of Units				Dimensions on Electromagnetic System of Units				Dimensions Length Mass and Time			Space Time System	
		Exponents only expressed												
		L	M	T	k	L	M	T	μ	L	M	T	L	T
Mass,	m	0	1	0	0	0	1	0	0	0	1	0	1	-1
Momentum,	mv	1	1	-1	0	1	1	-1	0	1	1	-1	2	-2
Moment of momentum,	mva	2	1	-1	0	2	1	-1	0	2	1	-1	3	-2
Energy,	mv^2	2	1	-2	0	2	1	-2	0	2	1	-2	3	-3
Force,	F	1	1	-2	0	1	1	-2	0	1	1	-2	2	-3
Spec. Induc. Capacity,	k	0	0	0	1	-2	0	2	-1	-1	0	1	-1	1
Magnetic Permeability,	μ	-2	0	2	-1	0	0	0	1	-1	0	1	-1	1
Electric Capacity,	C	1	0	0	1	-1	0	2	-1	0	0	1	0	1
Coef. of Self Induction,	L	-1	0	2	-1	1	0	0	1	0	0	1	0	1
Electrical Resistance,	R	-1	0	1	-1	1	0	-1	1	0	0	0	0	0
Electromotive Force,	E	½	½	-1	-½	3/2	½	-2	½	1	½	-3/2	3/2	-2
Magnetomotive Force,		3/2	½	-2	½	½	½	-1	-½	1	½	-3/2	3/2	-2
Electric Force,		-½	½	-1	-½	½	½	-2	½	0	½	-3/2	½	-2
Magnetic Force,	H	½	½	-2	½	-½	½	-1	-½	0	½	-3/2	½	-2
Electric Displacement,	D	-½	½	-1	½	-3/2	½	0	-½	-1	½	-½	-½	-1
Magnetic Flux Density or Induction,	B	-3/2	½	0	-½	-½	½	-1	½	-1	½	-½	-½	-1
Quantity of Electricity,	e	3/2	½	-1	½	½	½	0	-½	1	½	-½	3/2	-1
Quantity of Magnetism,		½	½	0	-½	3/2	½	-1	½	1	½	-½	3/2	-1
Total Magnetic Flux,		½	½	0	-½	3/2	½	-1	½	1	½	-½	3/2	-1
Planck's Constant,	h	2	1	-1	0	2	1	-1	0	2	1	-1	3	-2
Rydberg's Constant,	K	0	0	-1	0	0	0	-1	0	0	0	-1	0	-1
Newtonian Constant,		3	-1	-2	0	3	-1	-2	0	3	-1	-2	2	-1

In order to give a more comprehensive idea of what it means to have found the separate dimensions of specific inductive capacity and magnetic permeability Table I has been prepared. In the first column appear the common names of the kinds of quantities whose dimensions are given in the other columns. In the second column are given in some instances the symbols commonly used for the quantity, but there is no standardization of these symbols as yet, and these letters have no great significance. In the next group of four columns appear the common dimensions of the quantities on the electrostatic

system. For example, quantity of electricity has opposite to it the numbers 3/2 1/2 –1 1/2, as representing the exponents of the letters at the tops of these columns, which makes the dimensions of quantity of electricity, $L^{3/2}\ M^{1/2}\ T^{-1}\ k^{1/2}$, the same as the dimensions derived in the text above for the electrostatic system.

The next group of four columns gives the values of the exponents of L, M, T and μ, thus representing the dimensions of the same units or quantities on the so-called electromagnetic system of units. Since by Maxwell's equation, $k\mu = 1/c^2$, or $k = 1/\mu c^2$, we may convert any electrostatic dimensions into electromagnetic by merely substituting for the k in the electrostatic column its equivalent in terms of μ. And, because the k does not involve mass it is seen that the exponents of mass are exactly the same for each quantity in the two systems.

If for the k in the electrostatic column its equivalent dimensions, $L^{-1}T$, are substituted, the k is entirely eliminated, and the exponents given in the next group of three columns are obtained. Or, if for μ in the electromagnetic column its equivalent, $L^{-1}T$, is substituted, we obtain precisely the same exponents for L, M and T as by eliminating the k from the electrostatic column. This gives as a result a single system of units for all quantities in terms of length, mass and time only.

It appears from this new table that quantity of electricity and quantity of magnetism are quantities

of the same nature having the same dimensions as already pointed out. Also, specific inductive capacity and magnetic permeability have the same nature, as already stated. And again, electrical capacity and the coefficients of self and of mutual induction have the same dimensions. Electrical resistance becomes dimensionless in terms of length and time. It was already dimensionless in terms of mass in each of the former systems, and in the electromagnetic system it had the dimensions of a velocity times μ, which is now the reciprocal of a velocity. Hence electrical resistance is of the nature of the ratio of two different velocities, and has no dimensions at all, being simply a numeric like the quantity, β, which usually denotes the ratio between the velocity of an electron and the velocity of light.

Because of the relation expressed by Ohm's law, namely $R = E/I$, since resistance is dimensionless it follows that electromotive force has the same dimensions as electrical current and it so appears in the new table. Also magnetomotive force has the same dimensions as electromotive force, indicating that they are each quantities of the same nature. And again, electrical quantity divided by a time gives electrical current. We have seen that the rate of change of the quantity of electrical charge is equal to current. And again, electric force has the same dimensions as magnetic force, and electric displacement the same as magnetic flux density, that is magnetic induction.

In other words the results obtained by making k the reciprocal of a velocity are not irrational. The quantities that might be expected to be the same come out the same, and there is nothing about it that looks like chance. They do not come out in any haphazard fashion, as they certainly would if the dimensions assigned to k were different from the reciprocal of a velocity.

The last two columns of the table must be mentioned in advance of the narrative. In this new table appear only length and time, mass having been eliminated from the preceding columns. Reasons have appeared that make mass the reciprocal of specific inductive capacity, and taking specific inductive capacity as the reciprocal of a velocity, this gives mass the dimensions of a velocity, $L\,T^{-1}$. Hence, granting this, mass is not a fundamental unit like length and time, because it may be expressed in terms of them, and consequently it has been eliminated by putting for each M in the previous tables the dimensions $L\,T^{-1}$. This operation does not affect any of the statements just made concerning the similarity of the magnetic and the electrical units, for mass already appeared in each of these with the same exponent for each kind of unit in the two systems.

We have, therefore, arrived at a final table of dimensions of units in terms of length and time only, having eliminated three of the quantities that formerly were required, namely, specific inductive capacity, magnetic permeability, and mass. It is not

to be expected that we can go further than this, and it is very natural to retain space and time as the preferable kinds of quantities in which to express the dimensions of all units whether mechanical, electrical or magnetic.

III

It will be attempted in this section to give the reasons that have led to the belief that the dimensions of specific inductive capacity are the reciprocal of a velocity. Certain preliminary conceptions must, however, first be mentioned that may, perhaps, seem to lead us far astray from the point we are intending to discuss.

The first theory of atomic structure which had a definite form is that due to Sir J. J. Thomson, proposed at a time not long after his discovery of the existence of the electron. He postulated that each atom consisted of a sphere of positive electrification of fairly large dimensions, comparatively speaking, within which a number of electrons each having the same charge of negative electricity were circulating in orbits. His reason for assuming this positive sphere of electrification was undoubtedly for the purpose of retaining the negative electrons in a united system; for, it is supposed that negative electrons repel each other, and some means of counteracting this repulsion was required. This arrangement secured equilibrium for the circulating electrons.

At a subsequent time Sir Ernest Rutherford showed by means of his experiments on the scattering of the so-called alpha particles by matter that it is most probable that the positive charge of an atom cannot occupy the very large dimensions that Thomson assumed that it had, and that the dimensions are required to be very small indeed, even smaller than the dimensions of a single electron, and yet it was also thought that almost the whole of the mass of the atom resides in this very small positive nucleus as it is now called. Rutherford was supported in this view by the current form of electromagnetic theory as applied to either the atomic nucleus or to an electron. How this may be again requires a digression from the main topic before us, but it is very essential to understand what it is that electromagnetic theory has to say about this matter.

Several forms of electron have been proposed by different investigators, but the so-called "solid electron" due to Lorentz has received the most attention, and partly for the reason that it is more amenable to mathematical treatment than the other forms. At rest this electron has a spherical shape with a definite radius and the electrical charge is distributed throughout the whole volume of it. Each element of this charge is supposed to repel every other element, and the effect of them all together is to produce a very great pressure at the surface of the sphere tending to expand the volume of it. It is further supposed that this great pressure is counter-

balanced exactly by a sort of hydrostatic pressure exerted over the entire surface of the sphere. The mathematical treatment of the case by means of electromagnetic theory is not very simple and we shall have to limit ourselves here to the mere statement of some of the conclusions deduced from this form of electron.

It has been found, first, that the mass of the electron is not fixed and constant, but that it changes in value according to its velocity of translational motion. Moreover, that it differs from all other bodies that have ever been known before in that its mass is different when considered in the direction of its motion, the longitudinal mass, than it is when considered in a direction perpendicular to its motion, the transverse mass. However, the expressions for these masses as dependent upon the velocity show that there is very little change in the mass before its velocity is very great indeed, something like the tenth part of the velocity of light. For all values of its velocity less than this the longitudinal mass and the transverse mass have the same values approximately, namely

$$m = \frac{4}{5} \frac{e^2}{c^2 a},$$

where m is the mass, a the radius of the sphere, e the electrical charge of it, and c the velocity of light. The importance of this formula can scarcely be emphasized enough. Among other things it has led to

the belief that specific inductive capacity has the dimensions of the reciprocal of a velocity, which is the topic immediately before us.

Let us first see how this formula bears out the idea of Rutherford alluded to above. It is well known that the mass of the hydrogen atom, the lightest of all kinds of atoms, is very large by comparison with the mass of one electron, about 1846 times as great. If there is only a very small number of electrons in the hydrogen atom, as there is supposed to be on very good evidence, then it is obvious that practically the whole mass of the hydrogen atom must be concentrated in its positive nucleus, for there is nothing else in the atom. In a neutral hydrogen atom the charge of the positive nucleus is equal to the sum of the charges of the electrons in it, hence, the e of the above formula must be very nearly the same for the electron as it is for the nucleus. But the mass of the nucleus is very much greater as shown, and hence it must be that the radius of the nucleus, which is inversely as the mass, is very much smaller than that of the electron. All this, of course, is on the supposition that the whole of the mass of the atom is purely of an electrical origin as represented by this formula. This idea is substantiated the more we find out about all of these matters.

Now this result justified Rutherford in supposing that the nucleus of an atom is very small indeed, smaller than that of the electron itself, and this was

borne out by his experiment on the scattering of the alpha particles. It discredited the older Thomson theory which had the electrons immersed within the large sphere of positive electrification, thus making it many times larger in volume than the sphere of the negative electron. The Thomson atomic theory has accordingly been abandoned and the Rutherford theory taken its place. But with the abandonment of the Thomson theory went out also the benefit that it conferred of providing the means of securing stability for the electrons in orbits. Rutherford had no means of providing stability for the electrons in his new form of atom, but in spite of this his ideas have prevailed because they are supported by such strong evidence. There was the hope at least that something would appear to relieve the difficulty, even if electromagnetic theory as it then existed did not offer any solution of the problem.

It was known at the time that Rutherford proposed his theory that electromagnetic theory had already proved to be deficient in other directions. In particular Max Planck had made his startling proposal that the flow of energy probably takes place in what he called quanta, that is to say in multiples of a fixed minimum of energy, and this implied that energy partakes of a quasi atomic nature. This proposal was originally made on the strength of the known experimental facts connected with the radiation of energy by matter. Electromagnetic theory had proved itself incapable of giving results in agree-

ment with observation in this instance, and Planck did not hesitate to lay it aside. It is not proposed to enter into any discussion of these matters here more than to say that the ideas of Planck were receiving confirmation with a rapidity that probably exceeded his expectations as time went on, and they have now taken rank with other established physical facts irrespective of any difficulty in understanding them by means of electromagnetic theory.

But, to discredit electromagnetic theory at one point and still retain it at another is what we have really been obliged to do. This may seem to be a very questionable procedure at first thought to some, but it is not disconcerting when we take the proper view of the situation. Electromagnetic theory is still in the process of development as it has been since the beginning made by Maxwell. The original Maxwell equations account for most of the phenomena usually dealt with in the textbooks on Electricity and Magnetism as well as the more recent modifications of this theory, but they are not so general as the recent modifications of this theory, and do not account for certain other phenomena which the modern equations do account for. Here we have a precisely analogous case where the equations are useful for certain phenomena and are not to be trusted for other phenomena. In this instance the reasons why they are not to be trusted are quite apparent to us because we possess the better and more general forms of equations. Similarly, it is easily seen that

other phenomena might arise for which the best equations we now possess lead to error, and it may be expected that the reason for this will appear some time when still more general forms of equations are found. It would be ridiculous to throw away the best equations we now have, which already include a larger variety of phenomena than the earlier ones did, simply because cases have arisen that lie outside of the domain of the equations we possess. Hence it is necessary to learn to diagnose each case as it arises, and to decide whether the present equations are applicable to the case or not. This unsatisfactory state of affairs gives considerable latitude to the investigator, and the use of the word diagnosis is very apt, for to diagnose the case is precisely what the investigator at the present time is obliged to do in much the same sense as the physician makes his diagnosis by the symptoms presented.

In the present instance our diagnosis is that the Lorentz mass formula above given is correct, and applies to the case justifying the Rutherford form of atom, as showing that the nucleus of the atom is very small. Let us write down this equation once more as applied to the nucleus of the hydrogen atom. Denoting by e the electrical charge of a single electron, and assuming that there are two such electrons in every normal hydrogen atom in accordance with the author's theory, the charge of the hydrogen nucleus is equal to 2e being positive. The Lorentz formula then becomes

$$m_H = \frac{4}{5}\frac{(2e)^2}{c^2a_H} = \frac{16}{5}\frac{e^2}{c^2a_H},$$

the subscripts H referring to hydrogen. e and c are the same for hydrogen or any other atom, but m and a are different and require the subscripts. This formula may also be written interchanging the positions of m and a as follows:

$$a_H = \frac{16}{5}\frac{e^2}{c^2m_H}.$$

The numerical values of the quantities on the right of this equation are all known, and the numerical value of the radius may, therefore, be found. But, in this equation a good example is presented of the usefulness of dimensional formulæ, and both members of the equation should be examined to see that the dimensions are the same. Evidently the dimension of the left member on any system is simply a length, L. The dimensions of e^2 on the electrostatic system were derived above, and they may be taken from Table I, namely, $e^2 = L^3\,M\,T^{-2}\,k$, also those of $1/c^2m_H$ are $L^{-2}\,M^{-1}\,T^2$. Multiplying these together gives the dimensions of the right hand member of the equation as Lk, and not simply L as in the left member. The equation as it stands is, therefore, according to this sure test not a true equation between physical quantities according to any system of units. To make the dimensions of it agree we must multiply the left member by k or divide the right member by k as we prefer. This cannot of

course affect the numerical value, since the numerical value of k is unity on the electrostatic system of units, but it is most important to express the k, and not leave it to be understood that it is supposed to be there. It is the common practice among writers on electromagnetic theory to suppress the k as being unity and not affecting any numerical values on this account. Its omission, however, naturally leads one to the belief that the writers who suppress it suppose that it is dimensionless as well in terms of length and time. If our units of length, the centimeter, and of time, the second, are changed it will be shown presently that k is no longer equal to unity, and it becomes most important to include it in writing any formula. The revised equation including the k now becomes

$$a_H k = \frac{16}{5\, m_H} \left(\frac{e}{c}\right)^2.$$

The dimensions of each member are now L k on the electrostatic system of units. If, however, we give k the dimensions $L^{-1}T$, so as to transfer over to the new space-time system of units, the dimensions of each member become simply a time T, the L's cancelling out.

This suggests that we make a search for some constant quantity that has the dimensions of time simply, and compare this with the value of the time given in the above equation. The first constant that naturally comes to mind is the time of one revolution

of the electrons in the normal hydrogen atom. According to the author's theory of the atom explained in a later section the frequency of revolution is equal to twice the well-known Rydberg constant, which may be denoted by 2K, K being the Rydberg constant, numerically equal to 3.290×10^{15}, and 2K being 6.580×10^{15} revolutions per second. Therefore, the time of one revolution is 1/2K, which has the dimension of time simply, the dimension of the Rydberg constant being the reciprocal of a time.

Without the numeric 16/5 in the above equation, it is found that the reciprocal of twice the Rydberg constant is numerically closely equal to the literal part of it, and we obtain a new relation between physical constants,

$$2K = m_H \left(\frac{c}{e}\right)^2.$$

Both the dimensions of this and the numerical values agree. This enables us at once to write down a value for e^2/m_H, which may for the present be considered as the unknown part of the equation, as follows,

$$e^2/m_H = c^2/2K.$$

The numerical values of the velocity of light, c, and the Rydberg constant, K, as obtained from observations on spectra, are known with exceptional accuracy. The velocity of light is very close to the number 3×10^{10} centimeters per second, and the Rydberg constant very close to the number above

given, 3.290×10^{15}. Using these numbers we obtain a numerical value for e^2/m_H,

$$e^2/m_H = 1.36778 \times 10^8.$$

The decimal places are retained on the assumption that these are the exact values of c and K, and the result may easily be corrected for the very slight departure of c from 3 and of K from 329.

By substituting the value of 2K just given in the value of $a_H k$ previously given, we also obtain another important relation

$$a_H k = \frac{8}{5K},$$

thus connecting the radius of the hydrogen nucleus directly with the Rydberg constant, both members of this equation representing a time.

Having obtained a numerical value for the ratio of e^2 to m_H, each quantity might be found separately if there were any other experimental relation known between these two quantities. Fortunately there exists such another relation, in which the constants are known with accuracy. This is the constant obtained from the electrochemical equivalent of an element in electrolysis, and the constant may be referred to as the Faraday constant. The relation is

$$\frac{e \quad A_H}{c \quad m_H} = 9649.4,$$

where A_H denotes the atomic weight of hydrogen referred to oxygen as 16. Taking the ratio between the values of e^2/m_H above given and the e/m_H ob-

tained from this, after putting $A_H = 1.008$, and $c = 3 \times 10^{10}$, we obtain a numerical value for e, namely

19

$$e = 4.763 \times 10^{-10} \text{ electrostatic units,}$$

and using this numerical value of e, we get a numerical value for m_H, namely

$$m_H = 1.658 \times 10^{-24} \text{ grams.}$$

It is considered that these numerical values for both e and m_H are within the limits of error in the direct experimental measurement of these important constants. The best value of e obtained by Millikan by the use of his oil-drop method is 4.774×10^{-10}, and his value for m_H is 1.662×10^{-24}. By giving more attention to the best known values of c, K, and A_H the numerical values here given may be modified very slightly, but it is considered that these theoretical values of these constants will be the most reliable when these small corrections are properly attended to.

There is another experimental constant also known with considerable accuracy, from which a numerical value of the mass of the electron itself may be obtained. This is the ratio of e to cm_0, namely

$$e/cm_0 = 1.767 \times 10^7,$$

as determined by Bucherer, where m_0 signifies the mass of the electron at slow velocities. By the use of this and the previous value of e we find

$$m_0 = 0.898 \times 10^{-27} \text{ grams.}$$

IV

In this section we shall briefly give the reasons that have led to the belief that the dimensions of mass are those of a velocity. There are two equally forceful reasons why we have come to these dimensions for mass. The first is that a new expression has been found for Planck's constant, h, which demands that the dimensions of mass shall be those of a velocity, and the second reason is that the gravitational equation, which the author has developed, makes the same demand.[1] Thus far the constant, h, has made no appearance in the preceding expressions involving the constants, m_H, m_o, e, K, c and k, and it seems required to explain what is meant by this constant.

When energy is radiated by a gas, the amount of the radiated energy has been shown to be proportional to the frequency of vibration at which the radiation takes place. Or again, if light of a definite fixed frequency is allowed to fall upon a fresh metallic surface in vacuo, electrons are observed to be emitted from the surface, and the energy of each electron as it comes away from the surface has been

[1]A. C. Crehore, Physical Review, Vol. IX, No. 6, Sec. Series, June, 1917, equation (77), page 464.

found to be strictly proportional to the frequency of the light, and to depend upon nothing else. This relation was pointed out by Einstein, who proposed the very general formula, energy is proportional to frequency, and is equal to a constant, h, times this frequency. The ratio between the energy and the frequency is this constant quantity known as Planck's constant, h. Millikan has put this formula of Einstein to an experimental test, and has obtained surprisingly good results from his machine-shop-in-vacuo apparatus. A piece of metallic sodium is so held in a vacuum tube that a knife operated by a magnetic device outside of the tube can cut a thin chip off from the metal without destroying the vacuum in so doing. Moved into another position inside of the tube, light is allowed to fall upon the surface immediately while it is fresh, which causes electrons to be ejected from the metal. The velocity at which these electrons are ejected from the metallic surface was measured by means of an opposing voltage, and these voltages are proportional to the energy of the electrons, because energy is equal to the product of electromotive force and electrical charge, and the charge on the electron being constant the energy is proportional to the electromotive force. By employing light of definite known frequencies throughout a long range of wave-lengths, and by measuring the corresponding voltage for each frequency of the light, sufficient data was obtained for plotting a good curve. If the Einstein equation

represents a truth, the curve thus obtained by plotting the frequencies as abscissae and the voltages as ordinates ought to be a straight line. All the points obtained by Millikan in this experiment do lie remarkably near to the best straight line drawn by the method of least squares, and this throughout the whole of the range. This experimentally confirms the Einstein equation with considerable precision.

Now the slope of this line can be measured with considerable accuracy, too, the tangent of the angle representing volts divided by frequency, which is proportional to energy divided by frequency, but energy divided by frequency is Planck's constant, h. So a measurement of the tangent of the angle that this line makes gives a numerical value of h, assuming that the charge on the electron, e, is known. The best result of all of this work gave the numerical value for h, according to Millikan's statement in his book "The Electron,"

$$h = 6.56 \times 10^{-27}.$$

The new theoretical expression that the author has found for Planck's constant above referred to is

$$h = \left(\frac{a_H}{3}\right)^4 \frac{(2K)^3}{c},$$

the letters denoting the same quantities as they did above. The reason for believing that this is the correct expression for h is that both the dimensions of it and the numerical value are correct. Let us

first consider the dimensions. The dimensions of a^4 are L^4, of K^3, T^{-3}, and of $1/c$, $L^{-1}T$. All together they become

$$h = L^3 T^{-2},$$

which are seen to be the dimensions of h on the space-time system according to Table I. This expression introduces no doubt as to its dimensions because neither of the quantities k nor m are involved in it, and there is no doubt as to the dimensions of a, K and c.

According to the Einstein equation, which has been experimentally established, the constant, h, is equal to energy divided by a frequency, that is multiplied by a time. Energy has the dimensions of mv^2, mass times the square of a velocity on the common system of units, that is $L^2 M T^{-2}$, and this multiplied by a time gives the dimensions of h,

$$h = L^2 M T^{-1},$$

on the common system of units. This is of the nature of one of the mechanical units like force, energy, momentum, etc., in that it does not involve either k or μ. It has the exact nature of the quantity, moment of momentum, namely mva, the dimensions of which are evidently $L^2 M T^{-1}$.

If it is now assumed that the dimensions of h obtained above on the space-time system are exactly the same as the dimensions just given on the common system, we must have

$$h = L^3 T^{-2} = L^2 M T^{-1},$$

whence it immediately follows that the dimensions of M must be

$$M = L\ T^{-1},\ \text{a velocity.}$$

This is one of the reasons for considering that mass has the dimensions of a velocity. But before calculating the numerical value of h, let us examine the new expression from other points of view. The 2K, which appears in the expression, is equal to the frequency of revolution of the electrons in the hydrogen atom in its normal state, which accounts for the numeric, 2, satisfactorily. The only other numeric in the expression is the 1/3rd part of the radius of the nucleus. In connection with a spherical shape the factor 1/3rd of the radius occurs in the expression for the volume of a sphere.

An expression has been given above for the radius of the nucleus in terms of the Rydberg constant and specific inductive capacity, namely

$$a_H = 8/5Kk.$$

Using this expression for a_H in the new value of h above, we obtain as an equivalent expression for h, by eliminating a_H,

$$h = 8^5/15^4k^4Kc.$$

The dimensions of this expression are just the same, namely, $L^3\ T^{-2}$, if we regard k as the reciprocal of a velocity. But, the numerical value of k is unity, and the constant, h, is thus made to depend numerically upon the two constants K and c alone, each of which is known with exceptional accuracy. Adopting the

values as before, namely $c = 3 \times 10^{10}$, and $K = 3.290 \times 10^{15}$, we find

$$1/Kc = 1/3.290 \times 10^{15} \times 3 \times 10^{10} = 10.13171 \times 10^{-27}$$

and the numeric

$$8^5/15^4 = 32768/50625 = 0.647,269.$$

Multiplying these together gives h numerically as

$$h = 6.5579 \times 10^{-27}.$$

Upon comparing this value with the experimental value obtained by Millikan, as above described, namely, 6.56×10^{-27}, it is seen that the agreement is almost exact; and especially, if we should make c just a little under $3. \times 10^{10}$, which has not been done because there seems to be a difference of opinion as to the best value of the velocity of light among the different authorities, though all agree in making it under 3, we would obtain a yet closer value to 6.56. It is thus of considerable importance to make the numerical value of h depend upon only two constants that are so accurately determined as c and K. No doubt from this circumstance a more reliable value of h can be obtained than by any other method.

In examining the work of Millikan it is found that he has given some weight to the experimental value of h found by different methods by other experimenters, and has struck an average between them, finally adopting as a value that seems to fit all results the best of $h = 6.547 \times 10^{-27}$. Such a difference as that between 6.56 and 6.547 would make an easily perceptible difference in the slope of

the line that Millikan found as the best result of his experiment. It is the author's opinion that there was no increase in the accuracy of the value of h obtained by abandoning the true result of his own experiment because of the influence of the other experimental results.

The expression for h may be transformed again by means of the Lorentz mass formula, namely

$$a_H = \frac{16}{5m_H k} \left(\frac{e}{c} \right)^2$$

and the Rydberg constant

$$2K = m_H \left(\frac{c}{e} \right)^2$$

into the following form, in which K is eliminated,

$$h = \left(\frac{16}{15\,k} \right)^4 \frac{e^2}{m_H c^3}.$$

This form is of interest as connecting h with e and m_H. It is to be observed that the dimensions of the denominator are zero if we regard k as the reciprocal of a velocity and mass as a velocity, for mc^3 is the fourth power of a velocity and k^4 is its reciprocal. It follows that the dimensions of h are the same as those of e^2, and this is seen to be true according to the space-time table. There is, therefore, an intimate dependence of h upon e, and the fact that e is a constant charge would indicate that h must also be constant.

It seems as if the importance of these new thoughts can best be emphasized if we examine the case by supposing that our units of length and of time undergo a change and remain no longer the centimeter and the second. And, especially, the importance of retaining the k in all equations is well brought out. The second of time is not a natural unit as pertaining to the atoms. It depends rather upon the rotation of the earth on its axis. A more natural unit for time would be the time of one revolution of the electrons in the hydrogen atom. And the centimeter is not a suitable length, or rather a natural one, as applied to the atoms. A natural unit of length would be the distance travelled by light during the time that the electrons in hydrogen are making one revolution. Let us for purposes of illustration adopt these two units in place of the centimeter and the second.

The velocity of light then becomes unity instead of 3×10^{10} because light travels one unit of distance in one unit of time. Any velocity on the new system of units may then be obtained from the old units by dividing the numerical value of the old velocity by 3×10^{10}.

To convert length from one system to the other use may be made of the following:

A new unit of length $= 3 \times 10^{10}/2K = 4.559{,}25 \times 10^{-6}$ cm.
One centimeter (old) $= 2.1933 \times 10^{5}$ new units.
A new unit of time $= 1/2K = 0.151{,}975 \times 10^{-15}$ seconds.
One second (old) $= 2K = 6.58 \times 10^{15}$ new units.

To obtain the new values of other quantities, involving powers of L and T or both in the dimensional formulæ, multiply each L by $2K/3 \times 10^{10} = 2.1933 \times 10^{5}$, and each T by $2K = 6.58 \times 10^{15}$.

For example, twice the Rydberg constant becomes unity on the new system, for

$$2K = 6.58 \times 10^{15} = T^{-1}$$

on the old system becomes

$$2K = 6.58 \times 10^{15}/2K =$$

unity on the new system.

Specific inductive capacity is numerically unity on the old system, but, being the reciprocal of a velocity, the old value must be multiplied by 3×10^{10}, and this is, therefore, the value of specific inductive capacity on the new system. It is quite evident from this result that it will never do to omit to express specific inductive capacity in writing any formula.

Since mass has the dimensions of a velocity, the new unit of mass becomes 3×10^{10} grams, and represents a volume of water filling a cube some 31.07 meters on each edge. It is thus a very large unit for mass. The mass of the hydrogen atom expressed in the new units must, therefore, be a smaller number than it was on the old system by the factor 3×10^{10}. It was formerly 1.658×10^{-24}, and on the new system becomes

$$m_H = 1.658 \times 10^{-24}/3 \times 10^{10} = 0.55267 \times 10^{-34},$$

and

$$m_0 = 0.898 \times 10^{-27}/3 \times 10^{15} = 0.29933 \times 10^{-27}.$$

The expression above given for the Rydberg constant may be written

$$2Ke^2 = m_H c^2,$$

but, now, on the new system of units, $2K = 1$, and $c = 1$, hence numerically

$$e^2 = m_H = 0.55267 \times 10^{-24},$$

but this is not a dimensionally true relation. The K and the c must be expressed. The omission of them is exactly analogous to the omission of the k in the common system of units, and this example thus emphasizes the importance of the matter. The new value of e is the square root of the above, namely

$$e = 0.743 \times 10^{-12}.$$

If we examine the equation just given for the Rydberg constant, it is seen that $2Ke^2$ has the dimensions $L^3\ T^{-3}$, and so also does $m_H c^2$ have the same dimensions, and these dimensions are those of energy, the cube of a velocity on the space-time system. Numerically, therefore, since $c=1$, and $2K=1$, this energy has the same value as the mass of the hydrogen nucleus and also of the square of the charge on one electron. If we had always used this new system of units instead of the old system, and had omitted the c and K, one might be led to believe that the mass of the nucleus is just the same as its energy content, for we may regard this energy as the internal energy content of the nucleus. We know, however, that mass is a quantity of a differ-

ent kind from energy, and the use of the common system of units has never suggested that they are the same thing.

On the new system of units Planck's constant takes the very simple numerical value

$$h = \left(\frac{16}{15\,k}\right)^4,$$

and depends only upon the specific inductive capacity, because of the relation above, namely

$$h = \left(\frac{16}{15\,k}\right)^4 \frac{e^2}{m_H c^2},$$

for the second factor becomes unity because $c = 1$, and $e^2 = m_H$ numerically. Since $k = 3 \times 10^{10}$ on the new system of units, the numerical value of h becomes

$$h = \left(\frac{16}{15 \times 3 \times 10^{10}}\right)^4 = (0.355{,}55)^4 \times 10^{-40} = 1.598 \times 10^{-42}.$$

From this it appears that an accurate determination of the velocity of light alone will give a reliable value of h in the new system of units. But, to convert this over into the present C.G.S. system will also require a good value of Rydberg's constant. This is, in other words, the same result as obtained before, and makes h depend only upon K and c.

This section will be concluded with some reflections upon the unit of energy and the actual amount of energy contained in one gram of hydrogen. Energy, being the cube of a velocity, the new unit of

energy is c^3 times one erg, or 27×10^{30} ergs, a very large unit.

Using the common system of units, the energy represented by $m_H c^2 = 1.658 \times 10^{-24} \times 9 \times 10^{20} = 14.922 \times 10^{-4}$ ergs. This is equivalent to 1.4922×10^{-10} joules, since one joule equals 10^7 ergs, and this again is equal to the number of watts times the number of seconds, assuming that this energy can be extracted from the nucleus at some fixed rate. The total number of nuclei in one gram of hydrogen is equal to the Avogrado constant, 6.062×10^{23}, and the total energy in one gram is then $6.062 \times 10^{23} \times 1.4922 \times 10^{-10} = 9.0457 \times 10^{13}$ joules. Let us imagine that this energy may be extracted by some means at the rate of 1000 watts, or one kilowatt continuously. The supply of energy in the one gram will then last for

$$6.062 \times 10^{23} \times 1.4922 \times 10^{-10}/10^{3} = 9.0457 \times 10^{10}$$

seconds of time.

This is equivalent to about 2870. years' time. The enormous store of energy in the single gram of hydrogen is thus more forcibly impressed upon us when expressed in the familiar practical system of units.

V

The second reason for believing that mass has the dimensions of the reciprocal of specific inductive capacity is to be found in the author's theory of gravitation. A brief account of this theory is required in order to render the matter intelligible.

An equation[1] has been found which represents with fidelity all of the laws of gravitation as originally given by Newton. This equation carries the matter one step farther than Newton went, for he stopped with the ultimate particles of matter, and did not attempt to explain the cause of the gravitational force, contenting himself with the mere statement of the simple laws governing these forces. The equation referred to goes further and attributes the cause of the gravitational force to the electromagnetic action of the negative electrons contained within the atoms of matter. The equation attributes the existence of the force to the motion of the negative electrons, and shows that the positive nuclei of the atoms have nothing to do with the force except indirectly, because they are electrically bound to the electrons. If the motion of the electrons could by any means be stopped, it shows that the gravita-

[1] A. C. Crehore, Physical Review, Vol. XII, No. 1, Sec. Series, July, 1918, equation (10), page 17.

tional force would be reduced to zero, and while the motion persists, that the force is proportional to the kinetic energy that these electrons possess. This conception does not offer any prospect of ever being able to stop the motion of these electrons and thus to annul the gravitational force. If we were successful in stopping the electrons even to a small extent, at the same time we would destroy those forces that serve to bind the atoms together, and just so much matter would be disintegrated into individual electrons, and cease to exist as ordinary matter.

To some this conception of the gravitational force may at first present difficulty because it has already been seen that the mass of atoms is concentrated in the positive nuclei. This difficulty arises from an erroneous idea that must ever be combatted that mass and weight are the same thing. At a given locality on the earth's surface different masses are strictly proportional to their weights, but mass and weight are not by any means equal the one to the other. This is clear as soon as the body is carried to some different locality further from the center of the earth. Here the weight of the same piece of matter has changed, but no one believes that its mass is any different than it was before. It is necessary to distinguish carefully between mass and weight. They are entirely different concepts, but the reason that they are numerically proportional to each other is easy to see. It is because the electrical charge on the positive nucleus of each

atom in its common neutral state is exactly the same as the sum of the electrical charges on all of the negative electrons that it contains.

The equation that gives the average attraction of one electron for another, neglecting all of the electrostatic forces which are cancelled by the action of the positive nuclei of the atoms, is

$$F = \frac{1}{3} m_0 e^2 \beta_1^2 \beta_2^2 r^{-2},$$

where m_0 is the mass of the negative electron at slow velocities, and e its electrical charge. β_1 represents the ratio of the velocity of one of them to the velocity of light and β_2 that of the other. r is the distance between the centers of the orbits being described by each around its respective atomic nucleus.

It should be emphasized that the equation represents the average force for any pair of electrons. As the electrons revolve about their orbits the force that they exert upon each other evidently varies from point to point, and it is the average value for a large number of revolutions with which we are concerned resolved along the line joining the centers of the orbits. And, again, the planes of the two orbits may be inclined to each other by any angle whatever, which would produce a different force for each different inclination. An average is also required for the orientation of the orbits, because in any piece of matter save crystals it is probable that

there are pairs of orbits situated in every possible orientation. The above equation also includes such a space average as this, as well as the time average.

Let us now examine the dimensions of the two members of this equation. The dimensions of the left member are those of force, which on the common system of mechanical units are

$$F = L\ M\ T^{-2} \quad \text{(See Table I.)}$$

The dimensions of the right member are those of $m_0 e^2 r^{-2}$, since the betas are pure numerics without dimensions. On the electrostatic system as above derived the dimensions of e^2 are

$$e^2 = L^3\ M\ T^{-2}\ k.$$

Multiplying this by $M\ L^{-2}$, as the dimensions of $m_0\ r^{-2}$, gives the dimensions of the right member of the equation as

$$L\ M^2\ T^{-2}\ k = L\ M\ T^{-2}\ Mk.$$

The two members thus have the same dimensions only by making the dimensions of Mk equal to zero. By giving to mass the reciprocal of the dimensions of specific inductive capacity the equation for the force becomes a true physical equation between quantities having the same dimensions. This gives mass the dimensions of velocity, and makes force have the dimensions $L^2\ T^{-3}$. This is natural for the dimensions of force because energy is equal to force multiplied by distance, and multiplying these dimensions by L we have $L^3\ T^{-3}$, the cube of a velocity, which are the dimensions of energy in the table.

VI

Applying the gravitational force above given for a single pair of electrons to a large multitude of electrons such as must exist in any mass of matter, it becomes evident that the only quantities that change in the expression for the force, as one pair after another is included, are the speeds of the electrons, β_1 and β_2. Evidently, therefore, the whole attraction between two bodies of gross matter is expressed by summing up the β^2 over each body, the other quantities coming out as common factors in taking the sums. We have then, as representing the whole attraction between two gross bodies of matter, the equation,

$$F = \frac{1}{3} m_o \, e^2 \, \Sigma_1 \beta^2 \, \Sigma_2 \beta^2 \, r^{-2},$$

where r now represents the distance between their centers of gravity.

It is not possible to proceed further to get a numerical value of the force expressed by this equation until something is known about the values of the speeds of the electrons in the atoms. In order to arrive at a numerical value of these speeds let us now anticipate some of the results that come from the theory of the atom described in a later section.

If an electron revolves in a circular orbit with a frequency, n, it describes n circumferences in one second of time, that is, it travels a distance $2\pi an$ in one second, which, therefore, represents its absolute velocity, v. The theory of the atom, to which we are coming later, says that the frequency of revolution of the two electrons in the hydrogen atom is equal to twice the Rydberg constant, 2K. Hence their velocity must be

$$v = 4\pi a K, \text{ cm. per sec.}$$

And, again, the theory makes the kinetic energy of the two electrons together in the hydrogen atom equal to the product of Planck's constant and Rydberg's constant, h K. Hence,

$$m_o v^2 = h\ K.$$

But we have already obtained a value for Rydberg's constant, namely

$$2\,K = m_H\,(\frac{c}{e})^2,$$

and from these, by eliminating the K, we obtain

$$v^2 = \frac{h}{2}\,\frac{m_H}{m_o}\,(\frac{c}{e})^2 = \beta^2\,c^2.$$

Hence

$$\beta^2 = \frac{h\,m_H}{2e^2 m_o},$$

and

$$\beta = \frac{\sqrt{2}}{2e}\,(\frac{hm_H}{m_o})^{1/2}.$$

The above gives the speed of an electron in the hydrogen atom in terms of known constants. There are two electrons in a ring in this atom, and there is good reason to believe that the formula for the speed of any ring of p electrons may be obtained by putting p in the place of the 2 in the numerator above, thus giving the general expression for the speed of the electrons in any ring as

$$\beta = \frac{\sqrt{p}}{2e}\left(\frac{hm_H}{m_0}\right)^{1/2}.$$

Squaring this expression, and then multiplying the result by the number of electrons in the ring, the sum of the squares of the speeds of all the electrons in the ring is obtained. In the case of the hydrogen atom, where $p = 2$, this sum is

$$\Sigma_H\beta^2 = \frac{hm_H}{e^2m_0}.$$

When dealing with a single atom of hydrogen alone, having but this one single ring of electrons, this last expression represents completely the value of $\Sigma\beta^2$ required in the gravitational formula above given. Let us, therefore, write down according to these ideas the gravitational force on the average between just two hydrogen atoms at a great distance from each other. Since the two atoms are alike, the product of $\Sigma_1\beta^2$ and $\Sigma_2\beta^2$ is merely the square of the above value, and we evidently obtain as an expression for the whole force

$$F = \frac{1}{3} \frac{h^2 m_H^2}{e^2 m_o} r^{-2}.$$

The values of these constants are known, and hence the magnitude of the force may be calculated. But, if it is the same as the average gravitational attraction according to Newton's law, it must agree in magnitude with the following:

$$F = k' m_H^2 r^{-2}.$$

This is the force expressed by Newton's law on the average between two hydrogen atoms, where k' denotes the well known gravitational constant. Equating these two expressions for the same force, the m^2_H and the r^{-2} cancel, and the gravitational constant, k', becomes

$$k' = \frac{h^2}{3 e^2 m_o}.$$

The dimensions of this expression are correct, for by Newton's law $k' = F r^2/m_H{}^2 = L^3 M^{-1} T^{-2}$ on the common system, or $L^2 T^{-1}$ on the space-time system. By using the dimensions on the space-time system given in Table I for the quantities on the right of the equation, it will be found that they are the same as the dimensions of k' just obtained.

The numerical agreement between the experimental value of k' and the numerical value obtained by substituting the best values we possess for h, e and m_o is not as close as the agreement heretofore obtained between the theoretical values and the

experimental values above in the cases of e, h, m_H and m_o. The value of k′ obtained experimentally is about 666×10^{-10}, and the value of the right hand member above is 703.69×10^{-10} when we take $h = 6.558 \times 10^{-27}$, $e = 4.763 \times 10^{-10}$, and $m_o = .898 \times 10^{-27}$.

There are many difficulties presented in obtaining an accurate experimental value of the gravitational constant on the centimeter, gram, second system. Astronomers usually take the mass of the earth as unity in all astronomical problems, and the work of the astronomers alone is not sufficient to give a numerical value for k′. These measurements really come within the province of the physicist. The evidence for a greater value of k′ is so strong in the author's opinion that it is his belief that some day it will be possible to find where the present methods used for obtaining its numerical value require revision. The present value needs to be increased by about 5% to bring it into exact agreement with the theory. There is more confidence in the experimental numerical values of the other constants involved than there is in this one.

VII

It is pointed out in this section that there are two equivalent expressions for the mass of a body of gross matter, the one derived from the conception of weight and the other from the masses of the nuclei of the atoms. The force exerted by one body upon another according to Newton's law, the masses being denoted by m_1 and m_2, is

$$F = k' m_1 m_2 r^{-2}.$$

Upon substituting the theoretical value for k' just given this becomes

$$F = \frac{1}{3} \frac{h^2}{e^2 m_0} m_1 m_2 r^{-2}.$$

A third expression for the same force was given above in terms of the speeds of the electrons, as follows:

$$F = \frac{1}{3} e^2 m_0 \, \Sigma_1 \beta^2 \Sigma_2 \beta^2 r^{-2}.$$

Equating the last two expressions, the force is eliminated and we find

$$m_1 m_2 = \frac{e^4 m_0^2}{h^2} \Sigma_1 \beta^2 \Sigma_2 \beta^2,$$

from which evidently is obtained a general value for the mass of any body of gross matter,

$$m = \frac{e^2 m_0}{h} \Sigma\beta^2.$$

In this $\Sigma\beta^2$ is dimensionless, and e^2 has the same dimensions as h on the space-time system. Hence, the dimensions of m are the same as those of the mass of the electron, m_0, showing that the dimensions are correct. This equation tells us that the mass of a body is proportional to the sum of the squares of the velocities of all the electrons it contains, which is the same thing as the total kinetic energy of motion of all the electrons in the body.

Applying this to find the mass of a quantity of hydrogen gas consisting, say, of A atoms, it was pointed out above that the sum of beta square for one hydrogen atom is hm_H/e^2m_0. Multiplying this by A and substituting in the expression for mass just given, the e^2m_0/h cancels, and we obtain

$$m = m_H A,$$

which is evidently equal to the total mass of the gas.

Let us next look at mass from the standpoint of the masses of the atomic nuclei. If the charge of the nucleus is E, the Lorentz mass formula gives the mass of one atom as

$$m = \frac{4}{5} \frac{E^2}{c^2 ak},$$

where a is the radius of the nucleus, and m its mass. If the number of atoms in the body is A, many of

the atoms being of different kinds differing in charge and in radius, the total mass of the body M, may be expressed:

$$M = \Sigma_A \, m = \frac{4}{5c^2k} \Sigma_A \frac{E^2}{a}.$$

There is good reason to suppose that we should obtain the same value for the mass of the body from this as was obtained by the above method starting with the electrons, although the summation here expressed may be difficult to realize practically in many cases. If the formula is applied to hydrogen, however, the matter is simplified because the atoms are all alike, the charge on each nucleus being 2e, and every radius being the same, namely,

$$a_H = 8/5Kk = 3.2 \frac{e^2}{m_H c^2 k} = 4.8620 \times 10^{-13} \text{ cm.}$$

Substituting these values of $E = 2e$, and a_H in the last expression for M above, it reduces to

$$M = \Sigma_A m_H = \frac{4}{5c^2k} \Sigma_A \frac{(2e)^2}{a_H} = \frac{4}{5c^2k} \times \frac{4\, m_H c^2 k \, A}{3.2} = m_H \, A,$$

which is evidently equal to the total mass of the body of A hydrogen atoms.

We will not attempt at present to consider other atoms than that of hydrogen, that is to say, we will rest content without giving a general proof that the two different expressions for the mass of any gross body of matter are always equivalent.

VIII

Having developed a gravitational equation of great generality it is most natural to make every endeavor to test its validity at as many points as possible. The equation referred to is

$$F = \frac{1}{3} e^2 m_0 \Sigma_1 \beta^2 \Sigma_2 \beta^2 r^{-2}.$$

This may be applied to any two bodies of matter, simple or complex. We may, if we choose, apply it to the earth as one of the bodies and to a single atom on its surface as the other body, when the equation will give the weight of the atom. For simplicity let us deal with atoms at first having but a single ring of p electrons. The equation then takes the form

$$F = \frac{1}{3} e^2 m_0 \Sigma_p \beta^2 \Sigma_E \beta^2 r_E^{-2},$$

one of the velocity summations being taken over the single atom of p electrons and the other being taken over the whole of the earth, including every electron in every atom in the earth, and the distance r now becoming r_E, the radius of the earth.

It was pointed out above that the sum of the squares of beta for the hydrogen atom consisting of a ring of two electrons was

$$\Sigma_H \beta^2 = \frac{hm_H}{e^2 m_o}.$$

But the more general expression for a ring of p electrons is

$$\Sigma_p \beta^2 = \frac{p^2}{4} \frac{hm_H}{e^2 m_o},$$

which reduces to the above when $p = 2$. When the attraction of the earth for atoms of different kinds is written down, evidently the only quantity that has any different value for the different atoms is $\Sigma_p \beta^2$, for the properties of the earth remain fixed. If the above value for $\Sigma_p \beta^2$ is used, it is to be observed again that all of the quantities in it remain fixed except p, the number of electrons in the ring, assuming that there is a single ring atom only. It results from this that the weights of single ring atoms are approximately proportional to the squares of the numbers of electrons in the rings. Therefore, when the weight of any single ring atom is known, it is possible at once to write down the weights of all other single ring atoms. Assuming that the hydrogen atom has two electrons, and atomic weight 1.008 referred to oxygen as 16, the following weights for atoms having rings of p electrons are obtained.

p Electrons Per Ring	Weight of Ring or Atom
2	1.008
3	2.268
4	4.032
5	6.300
6	9.072

It should be remarked, however, that the strict proportionality between the square of the number of electrons and the weight is to be regarded as a first approximation only to the truth. The above formula for the speed of the electrons in rings makes the speed depend only upon the number of electrons in the ring, and is thus entirely independent of the radius of the ring. This is why it is regarded as a first approximation only, the value of the radius playing a minor part as affecting the speed, but yet a real one, and not exactly negligible. This formula is a radical departure from any previous theory of the atom. It contains within it the idea that the cause of the revolution of the electrons is the mutual action of the electrons in the ring upon each other.

According to electromagnetic theory there exists a tangential component of force exerted upon each electron in the ring due to the sum of all of the other electrons in the ring, and this force always acts in the direction of motion of the electron and is never zero at any speed, except the zero speed. And, if the motion should reduce to zero and then reverse going in the opposite direction, this force would also reverse and be in the direction of the motion again. According to this, unless there is some counterbalancing force, there must be a continuous increase of

the velocity of the electron, and no uniform velocity would ever be attained, for it is necessary that these tangential forces shall be zero as well as the radial force acting upon the electron before equilibrium is possible. It is supposed in electromagnetic theory that the counterbalancing force is that due to the action of the selected electron upon itself, but it is not possible to find what this force is without making certain assumptions with regard to the electron itself, which have the effect of begging the question, and, hence, no definite solution of the speed of electrons in rings has ever been obtained by means of electromagnetic theory.

This seems to be one of the cases where the electromagnetic theory we now possess is deficient. It is our diagnosis that the equations of electromagnetic theory as applied to rings of electrons are not correct, and the point where they require modification has been rendered evident by this gravitational theory. It will be pointed out in a later section that there is a factor, the so-called Doppler factor, in these equations that requires modification. It is not easy to make it clear here just what this factor means, but it arises from the introduction of the ideas sometimes referred to as retarded potentials, in which we have two different times to deal with, the time when the effect of the motion of the distant body arrives at the given body, and the time when it left the distant body. According to the recent work of Maga Nad Saha it has been pointed out that one

of these times should not be a time simply, but should depend upon the space coordinates as well, referring in fact to a generalized Minkowski space having four coordinates, x, y, z, and t.

The gravitational equation that we have been using was obtained by supposing that this Doppler factor is sensibly equal to unity, and the results obtained by making this supposition are the justification for it, especially since it has been pointed out by Saha that a change in it is demanded. If, now, we follow the same course in regard to the equations as applied to rings of electrons, then they lead to a definite value of the speed at which the tangential forces referred to above vanish, and thus afford a definite solution for the speeds of rings of electrons. However, the numerical values thus obtained are not to be trusted for the reason that there are other deficiencies in the electromagnetic equations, which will be referred to in a later section. These are also revealed by the gravitational equation.

After these remarks, which have led us somewhat away from the subject before us, let us return again to the weights of rings of electrons as given in the table above. Due to the suggestion contained in this table an attempt has been made to ascertain from the known weights of different kinds of atoms the particular combinations of rings of electrons which will add up to make the exact known weight of the atom in question. After many trials it has been found that, by slightly altering the weights in

the above table and making the weight of the rings as follows, the exact weights of a large majority of the atoms may be obtained.

p Electrons Per Ring	Weight of Ring
2	1.008
3	2.29643
4	3.99975
5	6.2895
6	9.137

TABLE A

	At. Wt. Observed O = 16 H = 1.008	At. Wt. Calc. O = 15.9990 H = 1.008	Total No. of Electrons	Arrangement in Rings in Atoms 2	3	4	5	6	% Error in Measured At. Wt.	% Difference in Calculated At. Wt.
H	1.008	1.008	2	1					0.0993	0.000
He	4.00	3.99975	4			1			0.25	—0.00616
Li	6.94	6.8893	9		3				0.144	—0.731
Gl	9.1	9.1371	6					1	1.10	0.408
B	11.0	11.0235	14	3		2			0.909	0.214
C	12.00	11.99926	12			3			0.0833	—0.00616
N	14.01	14.01587	16	2		3			0.0714	0.0376
O	16.00	15.9990	16			4			0.0625	—0.0061[illegible]
F	19.0	19.023	22	3		4			0.526	0.121
Ne	20.2	20.3115	23	2	1	4			0.495	0.552
Na	23.00	23.0228	26	3		5			0.0435	0.099
Mg	24.32	24.3112	27	2	1	5			0.0413	—0.0362
Al	27.1	27.0225	30	3		6			0.369	—0.286
Si	28.3	28.3109	31	2	1	6			0.3533	0.0387
P	31.04	31.0223	34	3		7			0.0322	—0.0571
S	32.06	32.0625	40	8		6			0.0312	0.00793
Cl	35.46	35.4488	29			1	5		0.0282	—0.0315
A	39.88	39.8953	43	1	3	8			0.0251	0.0384
K	39.10	39.054	46	7		8			0.0256	—0.1175
Ca	40.07	40.062	48	8		8			0.02495	—0.0199
Sc	44.1	44.062	52	8		9			0.227	—0.0867
Ti	48.1	48.062	56	8		10			0.208	—0.0800
V	51.0	51.053	58	7		11			0.196	0.1045
Cr	52.0	52.061	60	8		11			0.192	0.1178
Mn	54.93	54.919	61	4	3	11			0.0182	—0.0208
Fe	55.84	55.831	38	1				6	0.0179	—0.0165
Co	58.97	59.020	62	3		14			0.0169	0.0857
Ni	58.68	58.638	66	6	2	12			0.01705	—0.0717
Cu	63.57	63.613	68	3	2	14			0.01573	0.0683
Zn	65.37	65.317	69	3	1	15			0.0153	—0.0814
Ga	69.9	70.012	72	2		17			0.1431	0.1599
Ge	72.5	72.589	74		2	17			0.1379	0.1228
As	74.96	75.020	78	3		18			0.0133	0.0795
Se	79.2	79.185	80	2	2	15	2		0.1262	—0.0192
Br	79.92	79.905	68	1		4	10		0.0125	—0.0185

For example, it is supposed that the atom of Magnesium consists of the following combination of eight rings of electrons, 27 electrons altogether,

$$\begin{array}{llcl}
5 \text{ rings of four} & = 5 \times 3.99975 & = & 19.99875 \\
1 \text{ ring of three} & = 1 \times 2.29643 & = & 2.29643 \\
2 \text{ rings of two} & = 2 \times 1.008 & = & \underline{2.016} \\
 & \text{Total} & = & 24.3112
\end{array}$$

TABLE A (Continued)

Kr	82.92	82.917	89	4	3	18		0.01206	—0.00378
Rb	85.45	85.494	91	2	5	18		0.01169	0.0512
Sr	87.63	87.612	92	3	2	20		0.01141	—0.0206
Yt	88.7	88.900	93	2	3	20		0.1127	0.226
Zr	90.6	90.604	94	2	2	21		0.1104	0.00407
Cb	93.1	93.181	96		4	21		0.1074	0.0866
Mo	96.0	96.026	100	4		23		0.1042	0.0275
Ru	101.7	101.595	104	1	2	24		0.0983	—0.1033
Rh	102.9	102.883	105		3	24		0.0972	—0.0162
Pd	106.7	106.603	110	2	2	25		0.09375	—0.0912
Ag	107.88	107.891	111	1	3	25		0.00927	0.0103
Cd	112.40	112.306	115	2	1	27		0.00889	—0.0838
In	114.8	114.883	117		3	27		0.0871	0.0720
Sn	118.7	118.602	122	2	2	28		0.0842	—0.0826
Sb	120.2	120.305	123	2	1	29		0.0832	0.0876
Te	127.5	127.456	125		2	26	3	0.0784	—0.0346
I	126.92	126.938	109		2	7	15	0.00788	0.0145
Xe	130.2	130.288	131		1	32		0.0768	0.0680
Cs	132.81	132.795	140	3	6	29		0.00753	—0.0110
Ba	137.37	137.312	141	3	1	33		0.00728	—0.0420
La	139.0	139.015	142	3		34		0.0719	0.0112
Rare Earths.									
Ta	181.5	181.590	184	1	2	44		0.0551	0.0496
W	184.0	184.021	188	4		45		0.05435	0.0116
Os	190.9	190.878	193		3	46		0.05238	—0.0115
Ir	193.1	193.174	196		4	46		0.0518	0.0385
Pt	195.2	195.293	197	1	1	48		0.0512	0.0474
Au	197.2	197.308	201	3	1	48		0.0507	0.0550
Hg	200.6	200.613	206	4	2	48		0.04985	0.00650
Tl	204.0	204.020	208	4		50		0.0490	0.00965
Pb	207.20	207.292	209	1	1	51		0.00483	0.0443
Bi	208.0	208.052	216	8		50		0.04806	0.0249
Nt	222.4	222.315	227	4	1	54		0.04495	—0.0381
Ra	226.0	226.034	232	6		55		0.0442	0.0152
Th	232.4	232.331	239	6	1	56		0.0430	—0.0298
Ur	238.2	238.282	239		1	59		0.0420	0.0344

The measured atomic weight of magnesium is 24.32, and, if an error as great as one unit in the last decimal place has been made in the measured weight, the calculated weight comes within the experimental error.

It is not necessary to comment much upon this table, but it may be stated that the real test of the scheme is to be found with the elements of lower atomic weight. The table has been extended clear on through the periodic table of elements on the strength of the fact that the elements of low atomic weight seem to reveal that the atoms are largely built up of rings of four electrons. This is a matter which may be tested by means of the gravitational equation. How this may be will now be stated.

If we should take any piece of mixed matter made up of a large variety of atoms, the chances are that, if the atoms are really constituted as indicated in this table, the number of rings of just four electrons would greatly exceed any other kind of ring. By taking just one atom of each kind indicated by this table we would have a total of

1470	rings	of	four	electrons
185	"	"	two	"
86	"	"	three	"
35	"	"	five	"
7	"	"	six	"

thus indicating that the number of rings of four electrons greatly exceeds all others. If this is so, then the average speed of an electron in this mass of mixed matter must be very nearly the same as the

speed in a single ring of four electrons. Now, by means of the gravitational equation, the average speed of the electrons in the earth or in any other body may be found. If it comes out the same as the speed in a ring of four electrons, or nearly the same, this affords a test of the atomic weight table just given.

Let us first choose the earth for the test. Write down the attraction of the earth for a single hydrogen atom on its surface, both by means of Newton's law and by means of the gravitational equation, and then equate their values. The first is

$$F = k' \, m_H \, m_E \, r_E^{-2},$$

and the second is

$$F = \frac{1}{3} e^2 \, m_o \, \Sigma_H \beta^2 \, \Sigma_E \beta^2 \, r_E^{-2}.$$

Upon equating the r_E cancels, and we find

$$\Sigma_E \beta^2 = \frac{3 \, k' m_H \, m_E}{e^2 m_o \Sigma_H \beta^2}.$$

All of the quantities in the right member are known quantities, and the expression, therefore, gives us the sum of the squares of the speeds of all of the electrons in the earth. The numerical values are

$k' = 666. \times 10^{-10}$
$m_H = 1.662 \times 10^{-24}$
$m_E = 5.984 \times 10^{27}$ grams mass of earth.
$m_o = 0.90 \times 10^{-27}$

$$\Sigma_H \beta^2 = \frac{hm_H}{e^2 m_o} \text{ (see above) } = 0.531 \times 10^{-4}.$$

Hence we find

$$\Sigma_E \beta^2 = 1.825 \times 10^{47}.$$

Dividing this number by the total number of electrons in the earth must give the average square of the velocity of a single electron in the earth. The mass of the earth used above in grams is 5.984×10^{27}, and the number of electrons in each gram of matter is equal to the Avogadro constant, 6.062×10^{23}. Hence the number of electrons in the earth is approximately.

$$N = 6.062 \times 10^{23} \times 5.984 \times 10^{27} = 3.6275 \times 10^{51}.$$

Dividing the $\Sigma_E \beta^2$ by this number gives the average square of the speed of a single electron in the earth as

$$\beta^2_E = 0.503 \times 10^{-4},$$

and

$$\beta_E = 0.0071.$$

For comparison with this speed the numerical values of the speeds of electrons in rings according to the preceding formulæ are:

p Electrons Per Ring	β = the velocity in terms of the velocity of light
2	0.003641
3	0.005462
4	0.007283
5	0.009104
6	0.010925

The average velocity of an electron in the earth is, therefore, very close indeed to the velocity in a ring of four electrons, the comparison being 0.0071 to 0.007283. The velocity is thus much nearer to that of a ring of four than to that of any other ring. This supports in a very satisfactory manner the scheme revealed by the atomic weight table above.

IX

In the preceding example, by which the average speed of an electron in the earth was found, the mass of the earth in grams was used as a factor both in getting the total $\Sigma_E\beta^2$ and in getting the total number of electrons in the earth. Upon division to obtain an average the mass cancelled out. In other words, it was really not required at all, and no error introduced through an incorrect numerical value of the mass of the earth could affect the final result in any way. From this it is apparent that the result above obtained for the earth is very general, and applies to the planets and sun or any other form of matter just as well.

Let us consider two representative cases. First, apply the gravitational equation to find the attraction between a star, or large mass of nebulous gas composed entirely of hydrogen and nothing else, and a single hydrogen atom situated outside of the star at some distance, r, from its center of gravity. Denoting the mass of the star by M, Newton's law gives the average attraction as

$$F = k' \, m_H \, M \, r^{-2}.$$

According to the general expression for mass in terms of the sums of the squares of the speeds of

all the electrons within the mass above deduced, we have for the star

$$M = \frac{e^2 m_o}{h} \Sigma_M \beta^2$$

and for the hydrogen atom

$$m_H = \frac{e^2 m_o}{h} \Sigma_H \beta^2$$

whence by division

$$M/m_H = \Sigma_M \beta^2 / \Sigma_H \beta^2.$$

But, for the hydrogen atom having two electrons, the sum of beta square is twice beta square for a single electron, which may be denoted by β_H. Hence,

$$M/m_H = \Sigma_M \beta^2 / 2\beta^2_H.$$

And, solving for β_H^2, the preceding proportion is the same as

$$\beta^2_H = \frac{\Sigma_M \beta^2}{\frac{2}{m_H} M}.$$

In this form, since the star is by hypothesis composed entirely of hydrogen, the actual speed of an electron and the average speed are the same thing, all speeds being alike. Hence the equation really says that the average square of the speed is equal to the sum of the squares of the speeds of all the electrons in the star divided by $2M/m_H$. The average square of the speed must, however, be equal to the total sum of the squares of all the speeds

divided by the number of electrons in the star. It follows that the denominator of the last expression must be the total number of electrons in the star. This is evident directly, for the mass of one gram divided by the mass of the hydrogen atom must equal the number of atoms in a gram, namely $1/m_H$. Since there are two electrons per atom in hydrogen, the number of electrons per gram is twice the number of atoms, namely $2m/_H$. Since M denotes the number of grams in the star, evidently $2M/m_H$ must be the total number of electrons in the hydrogen star.

Now the number $1/m_H = 1/1.658 \times 10^{-24}$ is very nearly equal to the well-known Avogadro constant. Hence the number of electrons per gram of hydrogen according to the result is about twice the Avogadro constant.

The next representative case to be considered is that of a helium star attracting, say, a single helium atom at a great distance, r, from the star. We have a similar proportion to the above, namely

$$M/m_{He} = \Sigma_M \beta^2 / \Sigma_{He} \beta^2,$$

where the symbol (He) refers to the helium atom. According to the atomic weight table above given, the helium atom consists of a single ring of four electrons. Denoting the speed of one electron by β_{He}, the sum of the squares for one atom is

$$\Sigma_{He} \beta^2 = 4\, \beta^2_{He},$$

and the above proportion may be written

$$\beta^2_{He} = \frac{\Sigma_M \beta^2}{\dfrac{4}{m_{He}} M}.$$

In a similar manner to the case of the hydrogen star, the mean square velocity for the star must be the same as the square of the velocity of a single electron in the helium atom, β_{He}. Since the numerator represents the total sum of the squares of the speeds, it follows that the denominator must represent the total number of electrons in the star. And, evidently, $1/m_{He}$ is the number of helium atoms in one gram, and $4/m_{He}$ is the number of electrons in a gram of helium, and $4M/m_{He}$ must be the total number of electrons in M grams of helium, that is in the star.

Now, the mass of the helium atom compared with that of hydrogen is as the numbers 4.00 to 1.008. Hence, $4/m_{He} = 1.008/m_H$. Using the Millikan value of $m_H = 1.662 \times 10^{-24}$, we obtain

$$4/m_{He} = 6.065 \times 10^{23} = \text{No. electrons per gram of He.}$$

This number is very close to the well-known Avogadro constant, which is given by Millikan as 6.062×10^{23}. In deriving his value Millikan used a value of the atomic weight of hydrogen 1.0077, which he supposes is more accurate than 1.008. This will account for the difference in the last decimal place of the Avogadro constant.

These two examples have been chosen because the helium star, being based upon an atom with a

ring of four electrons, is typical of all the other kinds of atoms, that is, of any mixed mass of matter, while the hydrogen star forms an exception to the rule, as it does in other particulars.

Let us next consider an example of a planet, made up of any mixed kinds of solid matter, say,and compare the attraction of the planet for a distant helium atom with that of the helium star just considered for the same helium atom at the same distance. Let us also suppose that the masses of the planet and of the helium star are the same, each equal to M. Denote by A the Avogadro constant. As above, we have for the helium star

$$\beta^2_{He} = \frac{\Sigma_M \beta^2}{A\,M},$$

And, for the planet we have

$$\overline{\beta}^2 = \frac{\Sigma_M \beta^2}{x\,M}.$$

Here $\overline{\beta}^2$ is the average mean square of the speeds of all the electrons in the planet, and we have used x for the number of electrons per gram. Since we know that the masses of any two bodies are strictly proportional to the sums of the squares of the speeds of the electrons they contain, and since we have taken the mass of the helium star equal to that of the planet, it follows that the numerators above are equal. When we divide the one expression by

the other, these sums of squares of the velocities cancel out as well as the masses, and we have the simple relation

$$\frac{\beta^2_{He}}{\overline{\beta}^2} = \frac{x}{A} \text{ or } \overline{\beta}^2 = \frac{A}{x}\beta^2_{He}.$$

Now, it is well known that the number of electrons per gram, x, of any substance except hydrogen is approximately constant. This may easily be proved on either of the two suppositions that the number of electrons per atom are nearly proportional to the atomic number or to the atomic weight, it does not matter which, since the atomic weight is roughly twice the atomic number. Hence x is a constant number for any kind of mixed piece of matter like the planet we are considering. Moreover, it is believed that this number is very nearly equal to the Avogadro constant. If we make it equal to this constant, then the x and A cancel out above, leaving

$$\overline{\beta}^2 = \beta^2_{He} \text{ or } \overline{\beta} = \beta_{He}.$$

That is to say, the average speed of an electron in the planet is very nearly the same as the speed of an electron in the helium atom, and this is the same as the speed in a ring of four electrons.

This is, therefore, the reason why the average speed in the earth in the previous example came out the same as the speed in a ring of four electrons very nearly. The result is, therefore, general as ap-

plying to any planet, and the fact that it is so is intimately connected with the scheme given in the atomic weight table above, which makes the number of rings of four electrons greatly in excess of any other kind of ring. This arrangement may also be regarded as the reason why the number of electrons per gram is very nearly constant for all substances except hydrogen. These considerations seem to establish in a fairly satisfactory manner the scheme of the atomic weight table in this one particular at least, namely, indicating that rings of four electrons greatly preponderate in the atoms of different elements.

X

The equation that has been given above as representing the average gravitational force between two revolving electrons has shown, so far as we have gone, that the force is directly proportional to the product of the masses and inversely proportional to the square of the distance, and that the magnitude of the force is in approximate agreement with the true gravitational force. There is one more feature of the law with which it must agree before we can accept it as representing the complete force, and this is the fact that crystals, too, are known to obey the same law as other kinds of matter. That is to say, the weight of a crystal is independent of the way we hold it with respect to a plumb line, or the attraction of two crystals for each other is independent of the orientation of their axes relative to each other. These facts have been fairly well established experimentally, and the gravitational theory must show how this can be possible.

The gravitational equation, which has been used thus far above, was derived on the supposition that the orbits of the two electrons, with which it deals, are turned in every possible way with respect to each

other, and an average has been taken. In a crystal, however, the axes of the electrons cannot be assumed to be turned in every possible way with respect to each other. We are, therefore, obliged to go back to the original equation, from which the one we have been using was derived, to find an answer to this phase of the subject. The equation in question is

$$F_r = \frac{1}{3} m_0 e^2 \beta_1^2 \beta_2^2 r^{-2}.$$

But this in turn came from taking the space average of the following equation, which represents the time average of the force for one fixed position of the two orbits only, namely,

$$F_r = \frac{1}{2} e^2 m_0 \beta_1^2 \beta_2^2 \left[1 - (-X \sin \alpha + Z \cos \alpha)^2\right] r^{-2}.$$

In this equation the X, Z and α serve to fix the positions of the two orbits in space relative to each other. The equation represents the force of attraction that the second electron exerts upon the first on the average, this average being taken for a large number of revolutions of both electrons in these fixed orbits, the whole force being resolved along the line joining the centers of the two orbits, this being the fixed distance, r. The X and Z are the direction cosines of the position of this center line referred to three rectangular axes, i, j and k, passing through the center of the orbit of e_1, the first electron. A reference to Fig. 1 may serve to make this clear.

The orbit of the first electron, e_1, is shown in perspective as an ellipse lying in the horizontal plane, ABCD. The instantaneous position of the electron in this orbit is shown at e_1 revolving about the center, O_1. The three rectangular axes, i, j and k of unit length along which directions the distances x, y and z are measured, as representing the co-

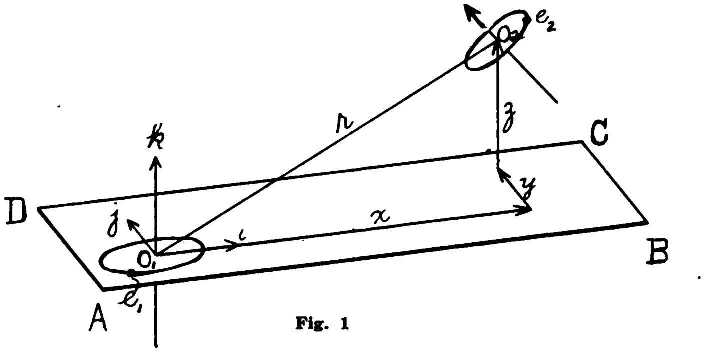

Fig. 1

In the above figure the orbit of the first electron, e_1, is shown in perspective as an ellipse, while the instantaneous position of the electron in shown at e_1 revolving about the center O_1. The arrow through the center of the orbit e_2 at O_2 is supposed to be perpendicular to the orbit of e_2, which is shown at the right of the figure also in perspective as an ellipse.

ordinates of the position of the center of the orbit of e_2 are shown having their origin at O_1. The direction of the k or z-axis is perpendicular to the plane of the orbit, namely, ABCD. The arrow through the center of the orbit of e_2 at O_2 is supposed to be perpendicular to the orbit of e_2, which is shown at the right of the figure also in perspective as an ellipse.

This plane in general intersects the plane of e_1 when produced in some line, and the direction of the j or y-axis is chosen so as to be parallel with this line of intersection. So the direction of the i or x-axis is also determined, this being perpendicular to both the j and k, or the y and z-axes. The i and j-axes thus both lie in the plane ABCD.

The angle α is that between the axes of the two orbits, or, which comes to the same thing, that between the planes of the two orbits. From this definition of the i, j and k-axes it is evident that the direction of the k-axis alone remains fixed for any and all positions of the orbit of e_2. If the position of the orbit of e_2 is changed without changing the position of its center the i and j axes may be made to rotate around in the plane ABCD because the j-axis is always parallel to the line of intersection of the two orbits.

Briefly, it may be stated that the last equation above becomes exactly equal to the preceding, if the numeric 2/3rds is substituted for the whole bracket in the last equation. This numeric 2/3rds is obtained as an average value if we give to the orbit of e_2 all possible positions at one fixed distance, r, between the centers of the orbits, the locus of the center of e_2 describing a sphere with O_1 as center, and at each point of this sphere the orbit of e_2 takes all possible orientations.

The first equation may, therefore, be used in general for the average force between the two re-

volving electrons when their axes may be considered to have an average or mean position, but the second equation must be used when the directions of the axes are fixed and specified as they are in a crystal.

And again, it may be stated that, if we put in place of the single orbit e_2 four different orbits having their axes inclined to each other in accordance with the four medial lines of a regular tetrahedron, then the average force due to the group of four electrons is fixed and constant, and is independent of the orientation of the group of four electrons, provided they preserve their relative directions among each other. Moreover, the average force due to a single electron in the group of four is obtained by giving the bracket in this equation the value 2/3rds as before.

It follows from this that any two bodies made up of such groups of four electrons will behave exactly the same as any other two bodies, and be independent of the orientation. Now, this is exactly the arrangement of the axes of the atoms in the class of crystals belonging to the cubic or isometric system. This can be shown quite independently of the particular form of electromagnetic theory applied to the case, and is, therefore, probably true. It is proved by assuming that there is a tendency on the part of any atom to turn the plane of rotation of every other atom until it comes into parallelism with itself, when the turning moment of force vanishes. Coupling this principle with the known locations

of the centers of the atoms in these crystals, it follows that they are grouped in four equal quantities, and that each group has its axis parallel with one of the medial lines in a regular tetrahedron.

This class of crystals, therefore, satisfies the condition that the weight is independent of the orientation of the axes of the crystal according to the above equation. Nothing can, as yet, be said of the other classes of crystals, and so there must be left a single exception to the law simply because we do not know anything about the directions of the axes of the atoms in these crystals as yet. When it is shown that all crystals may thus be divided into four equal groups of electrons, the law will be general in every particular.

XI

In this section we shall consider more in detail the source from which this gravitational equation was derived. It has not been derived directly from the current form of electromagnetic equations, but these have required some modification. The surprising thing is that the modification required is so little. It may be said that the above equation would never have been obtained without the use of the present form of the electromagnetic equations. This equation was obtained by taking the time average of a much longer equation, that which represents the instantaneous force between the two electrons when they are in certain positions in their orbits. Each averaging process simplifies the equation. For example, there are no terms in the above equation that refer to time. The time disappeared in taking the time average, and contributed only to the numerical coefficient. So, again, in the space average later, the terms containing X, Z and a disappeared and entered into the numerical coefficient, giving a very simple final result.

In the original equation for the instantaneous force there appears in the coefficient multiplier of the equation the so-called Doppler factor,

$$A = \frac{\delta t}{\delta \tau} = 1 - \frac{q_2.R}{C R}.$$

This appears as $1/A^3$ in the coefficient of the equation. Now, when the speed is very small as compared with the velocity of light, the second term of A becomes very small, and the value of A approaches unity when the speed approaches zero. In deriving the time average of the instantaneous force, the author assumed that this factor was sensibly equal to unity. With this assumption the equation obtained is not exactly the same as that given above, but is exactly the same, if we omit the factors $m_0\beta_1^2$. To omit these quantities means that the magnitude of the force is very different, since m_0 is equal to $.898 \times 10^{-27}$, and β_1^2 of the order of 10^{-4}. Without these factors the derived force was, therefore, this great number of times larger than the gravitational force. This result is not only absurd, but it is fatal to the present form of the electromognetic equations, for we are absolutely certain that no such great force exists at these large distances. If the theory predicts them, the theory is erroneous.

Moreover, the omission of the β_1^2 is just as bad as the omission of the m_0, although it does not affect the magnitude so much, because its omission means that one body must attract the other with a different force from that exerted by the other body on the one. This is contrary to one of the best established laws we possess, due to Newton, that of equal action and reaction.

There is justification for the inclusion of both of these factors, and the fact that we obtain an equation that represents the gravitational laws faithfully in all respects when we do include them is additional justification. The justification for including the β_1^2 is that it makes the force conform to the established law of equal action and reaction. The justification for including m_0 is that without it the dimensions of the equation are not correct, and the quantities on the right of the equation do not represent a force. The dimensions of force are

$$F = L\,M\,T^{-2},$$

and the dimensions of the right member without the m_0 are on the electrostatic system of units,

$$L\;M\;T^{-2}k,$$

which cannot be the same as force. On the space-time system, putting $k = L^{-1}T$ and $M = L\,T^{-1}$, these become respectively

$$F = L^2\;T^{-3}$$

and

$$L\;\;T^{-2}\;.$$

These last dimensions require to be multiplied by a velocity in order to make them equivalent to a force. That is to say, they must be multiplied by a mass. Since m_0 is a mass, and since its numerical value when coupled with the β_1^2 brings the magnitude of the force into agreement with the gravitational force, we are justified in believing that the original equations were defective in not including such a quantity. It is most significant, too, that the numer-

ical value of the mass required to make this correction should have a value equal to the mass of the electron itself. It ought to be directly related to the electron if we are on the right track in getting at the origin of the gravitational force.

It must be stated here that the author has been taken to task in a long article by G. A. Schott for making what he considers an error in the derivation of the average force from the original electromagnetic equations of Lorentz. The error consists in assuming, as was done, that the Doppler factor above referred to is sensibly equal to unity. Fortunately Schott has checked the original equation for the instantaneous force with which we now begin, and finds the same result. And again, he has checked the equation that the author obtains as the time average of the force on the assumption that this Doppler factor is equal to unity, namely, the equation given above omitting the m_0 and the β_1^2. He has gone further and has obtained a time average assuming that this Doppler factor is variable with time. The result that he thus obtains does not resemble the gravitational law in any other respect than one. This is that the force is inversely as the square of the distance. The fact that we obtain an equation that represents gravitation so exactly in the manner that has been described above is of considerable interest in itself, especially in view of the fact that it is known that the present form of the Lorentz equations is deficient in certain cases. It is our diagnosis

that this is one of the cases where it requires modification, and we are particularly fortunate to have been led so near to a result which expresses the facts by sticking to the equations as far as we have without the necessity of modifying them.

Since this work was published Mega Nad Saha has derived a new form of electromagnetic equations based upon considerations of the general Minkowski space, and in accordance with the theory of relativity. His results show that the $\delta\tau$, which appears above in the expression for the Doppler factor, should be regarded as the differential of a generalized Minkowski space coordinate. This involves x, y, z and t, and does not depend upon time alone. In other words, reasons have already appeared from a quite independent source why there must be some modification in the Lorentz form of electromagnetic equations. It seems necessary to the author that these changes will eventually show that some such factor as m_0 will have to be introduced into the original equations. Also that a similar factor will appear in the so-called Doppler factor, the second term of it, which will result in its having a value that approximates unity very closely indeed.

The result of Schott, taking this factor as given above, will therefore prove to be of little value. It gives no useful result as it stands.

The above explanation has been offered for the reason that among physicists this criticism of Schott has been widely circulated. It is believed that it is fully met.

XII

In several places in the foregoing sections reference has been made to the author's theory of the atom. A brief account only of this theory will be given in this section, and for a more detailed description the reader is referred[1] to the author's book "The Atom." Use is made of electromagnetic theory in part and also of the Einstein equation connecting energy and frequency by a constant ratio, namely Planck's constant, which has been referred to above.

When an electron revolves in a circular orbit, as it is supposed to in the normal undisturbed state of all atoms, it exerts a certain mechanical force upon the stationary nucleus of an atom at a great distance away. It may be shown by means of the Lorentz form of electromagnetic theory that the mechanical force thus exerted by the revolving electron upon the distant atomic nucleus, due to its motion alone, is a circular force, provided the plane of the orbit of the revolving electron is perpendicular to the line joining the centers of the stationary nucleus with the center of the orbit. That is to say, this mechanical force may be represented by a vector having a constant magnitude but revolving at a uni-

*[1]D. Van Nostrand Co., New York, 1920.

form rate around the stationary nucleus as a center. It may thus be represented by an imaginary circle with the nucleus as a center, the plane of the circle being parallel to that of the orbit of the distant revolving electron, and the frequency of the revolving force the same as that of the revolving electron.

Moreover, this mechanical force is proportional to and in the opposite direction to the acceleration of the revolving electron. When an electron revolves in a circular orbit at a uniform rate the acceleration of it has a value constant in magnitude only, but continually changing in direction, since it must always be directed towards the center of the circle in which the revolution takes place. The acceleration of the electron may then be represented by a vector having the opposite direction to the position of the electron itself, namely the position vector. Both vectors, the position vector and the acceleration vector, always remain in opposite directions as they each revolve at the same rate, and thus describe circular paths. The direction of the force exerted upon the distant atomic nucleus is therefore in the same direction as the position vector, and opposite to that of the acceleration.

If, now, the plane of the orbit of the revolving electron is turned so that it does not occupy a plane perpendicular to the line referred to joining centers, then the electromagnetic theory tells us that the mechanical force exerted upon the distant stationary nucleus is proportional to that portion only of the

acceleration which is resolved in the plane perpendicular to the line of centers, and that it is always confined to a plane perpendicular to this line. Or, in other words, the plane in which the mechanical force is exerted is invariable, always being in a plane perpendicular to the line of centers, even though the plane of the orbit is not. And, again, the component of the acceleration of the revolving electron which is in the direction of the line of centers has no effect in producing any force upon the stationary nucleus.

These statements may be easier to comprehend by reference to Figs. 2 and 3. Let O be the position of the fixed atomic nucleus, say that of an atom in the photographic plate receiving the radiated energy from an electron in the distant hydrogen gas. Let e represent the instantaneous position of this revolving electron in the hydrogen atom describing a circular orbit represented in perspective in the figure about the fixed center, O′. To render the positions of these fixed points and planes referred to more apparent, use is made of a sort of box framework which is for no other purpose than to assist the eye in locating these geometrical objects in space by means of a figure drawn in a plane.

The fixed nucleus is located in the center of the plane PQRS at the left end of the box. The orbit of the revolving electron is shown by the small ellipse, as representing a circle in perspective on the right end of the box in the plane P′Q′R′S′, which is parallel to PQRS, about the center O′. These two planes

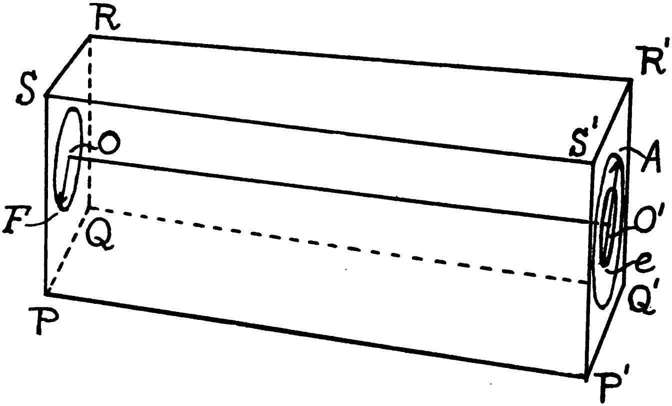

Fig. 2

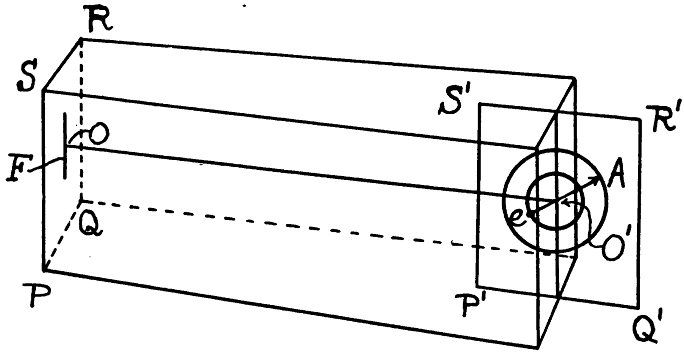

Fig. 3

The box-like framework is used to assist the eye in locating the geometrical objects in space. O represents the position of the fixed atomic nucleus and e, the instantaneous position of this revolving electron in the hydrogen atom. The circular orbit which it describes is represented in perspective about the fixed center O′.

are each perpendicular to the line OO′ joining centers. The acceleration of the revolving electron is then represented by the revolving vector O′A, which has the opposite direction to the position vector O′e. In this case the mechanical force exerted by the electron e upon the stationary nucleus O is represented by a circle in the plane PQRS about O as center, and the instantaneous position of the force vector, OF, is opposite to the acceleration O′A, and the same as the position vector O′e.

If, now, the plane P′Q′R′S′ is turned around through a right angle, as represented in Fig. 3, carrying with it the orbit of the revolving electron, e, then the mechanical force exerted by it upon the stationary nucleus at O is still in the same plane, PQRS, but it is not now represented by a circle but by a vertical straight line in this plane. The instantaneous position of the force vector is OF, which describes a harmonic motion up and down this vertical line. For any position of the plane P′Q′R′S′ intermediate between the parallel position as in Fig. 2 and the perpendicular position as in Fig. 3, the force-vector still remains in the same plane, PQRS, but is then represented by an elliptical motion in this plane about O as center, the circle in the first instance and the straight line in the second being but particular cases, the limiting cases, of elliptical motion.

If we should view the orbit of the revolving electron from the stationary nucleus, it would appear

as an ellipse in general when the orbit is not perpendicular to the line of centers. This ellipse represents the path of the position vector of the electron resolved in a plane perpendicular to the line of centers. Similarly, the circle representing the acceleration of the electron would appear as an ellipse when viewed from the same point, and this ellipse represents the resolved portion of the acceleration in a plane perpendicular to the line of centers. The mechanical force upon the distant nucleus is proportional to this resolved portion of the acceleration but in the opposite direction, and, hence, the mechanical force exerted upon the nucleus is an elliptical force, still, however, being in a plane perpendicular to the line of centers.

These ideas may be applied to obtain the effect of several electrons simultaneously acting upon the same stationary nucleus. If it is supposed that the selected stationary nucleus represents just one typical atom, say, in a photographic plate receiving the radiated energy from a distant mass of gas confined in a vacuum tube, then it may be supposed that the record produced upon the plate after development will depend entirely upon the radiated energy received by the atomic nuclei of the plate, and that what is received by one atom is also received by another. It is also to be inferred that the energy received by each nucleus is directly related to the mechanical force acting upon the nucleus during the reception of the radiated energy, but this, as we have

shown, depends upon and is proportional to the acceleration of the revolving electron in the distant gas. The total force upon the nucleus is proportional to the total vector sum of the accelerations of all of the revolving electrons in the mass of gas resolved in the plane referred to, whose radiated energy reaches the photographic plate and the stationary nucleus of an atom in the plate.

Assuming that there are two electrons in the normal state of each hydrogen atom situated at the opposite ends of a common diameter, the sum of the accelerations of the two electrons in one atom is evidently zero, for each is represented by a revolving vector of equal magnitude and opposite direction to that of the other, the geometrical sum of the two vectors being zero. If this is true of one atom it is true of them all. Hence, it follows that the mechanical force exerted upon the distant stationary nucleus in the photographic plate is zero for each hydrogen atom independently, and therefore, is totally equal to zero. The plate will show nothing upon development after exposure to a mass of hydrogen gas in its normal condition. This is in entire agreement with the observed facts, and this also explains in a satisfactory manner why the total radiation from a normal mass of hydrogen gas is zero.

It is not so easy to explain this matter if we should assume that each hydrogen atom has but a single electron in accordance with the Bohr theory of the atom. In such a case the force due to each

atom independently would not be zero, and whether the total force due to a multitude of such atoms in a mass of hydrogen gas is completely zero or not must depend upon the statistical question how completely the forces due to different atoms annul each other on the average. The chances that they will annul each other seem to be considerably greater if it could be assumed that the radii of the orbits of the electron in every atom is the same, and that the frequencies of revolution are also the same in every atom. But, the Bohr theory of the atom does not make these radii and these frequencies the same, and the probability that there will be a complete cancellation of the resulting force upon the stationary nucleus is thereby made far more improbable than it would otherwise be.

No attempt has been made to solve this statistical problem so as to show that the radiation from the Bohr hydrogen gas should be zero according to electromagnetic theory, because the matter has lost its interest. The two electron atom fills all the requirements perfectly not only in this respect but in every other respect that has been presented.

It may now be shown that the theorem above mentioned as to the mechanical force due to a revolving electron upon the stationary nucleus of the distant atom is perfectly true according to electromagnetic theory for any kind of motion of the revolving electron, whether it follows a circular path or not. That is to say, the theory shows that the mechanical

force is proportional to the resolved acceleration no matter what its character may be. A little reflection will show that this is a direct result of the theorem as established for simple circular motion. Suppose, for example, that two independent circular motions are geometrically added together. They may each have different radii and periods. The resulting curve may be considered to represent the complex path followed by the electron in question. The resulting mechanical force due to an electron thus moving would be the same as if two electrons were revolving in circular orbits of the radii and frequencies above mentioned, and, since each of these two component forces is proportional to the resolved acceleration of the electrons respectively, the resulting force is just the same as if the single electron moved in the curve compounded of the two circular motions.

Now, any motion of an electron whatever represents a certain acceleration, and any acceleration whatever may be resolved into an infinite series of simple circular accelerations according to Fourier's well-known theorem. Each of these component circular accelerations contributes its proportional mechanical force upon the distant stationary nucleus, and the total force is proportional to the sum of the component accelerations resolved in the perpendicular plane. The sum of the component accelerations is again equal to the actual acceleration in any complex path whatever that the electron may follow.

The total mechanical force is then proportional to the actual resolved acceleration.

The next step in the process of arriving at an atomic theory is to go to the photographic plate to observe what effect is produced upon it when the hydrogen gas is not in its normal condition, but when it is caused to radiate energy due to being bombarded by electrons from some external source. By the use of a diffraction grating or of a prism it is possible to sort out, as it were, the component frequencies that exist in the radiated energy, and to obtain an exact measure of these frequencies. The result which has been experimentally obtained from hydrogen is that the energy radiated is always emitted at some definite frequency, and that no other frequencies have ever been observed by the use of hydrogen gas than those expressed by the simple mathematical formula

$$\nu = K\left(\frac{1}{\tau_2^2} - \frac{1}{\tau_1^2}\right).$$

In this formula K represents the important Rydberg's constant, which has been discussed in the foregoing sections. It not only appears when hydrogen is employed but also when other elements are used, but in a different way in each case. The ν represents the frequency of the received energy, and the τ_2 and τ_1 stand for integers, 1, 2, 3, etc. Any values of τ_2 and τ_1 may be substituted in the formula, and the resulting frequency calculated will be found

to be emitted by the hydrogen gas under the proper conditions. The only restriction is that those values of these integers which make the frequency come out with a negative sign are prohibited, no meaning being attached to a negative frequency. Moreover, no other frequencies than those given by this formula are ever observed by the use of hydrogen.

It may be inferred immediately from these facts that the force acting upon the selected stationary nucleus of an atom in the plate always has the same frequencies, and then, by means of the theorem just given, that the sum of the resolved accelerations of the electrons in the hydrogen gas emitting the radiation contains these frequencies and no others. This gives the first intimation regarding the component frequencies in the acceleration of the motion of the electrons of the gas that we have possessed. But, because it is merely a sum of the effects due to a multitude of atoms, it is not as satisfactory as if it told something about the acceleration of an individual electron.

It shows, however, that it is possible that the frequencies emitted by hydrogen are due entirely to the character of the motion of the electrons in the gas, that is to say, to the form of the orbits followed by the electrons, when disturbed and compelled to depart from their normal circular motion. It is well known that, as soon as the cause of the disturbance is removed, the electrons quickly return to their normal condition again and cease to radiate energy.

What happens to an individual atom may be pictured by imagining that the electrons are driven out from their circular orbits, one of them going out to some maximum distance from the nucleus, and then returning again after the disturbance is over. The paths, by which these electrons return to their original orbit, cannot possibly be simple circular paths, and, unless the paths are circular, their accelerations cannot possibly be simple circular accelerations. Any other form of path demands more than one simple circular acceleration, which is the same as saying that these quasi spiral paths demand more than one frequency of vibration. And, again, since the motion ends finally in the original circular orbit from which it started, the case of a single atom demands that a series of frequencies shall be emitted at once in one operation of the two electrons, and that the frequency shall gradually fade away and become zero when the final orbit is attained. For, in this orbit the radiation has no frequency. As the periods of the two electrons approach equality in attaining this orbit, their difference, or vector sum, becomes less and less until it vanishes altogether.

It is possible to write the equation above given for the frequencies emitted by the hydrogen gas as follows:

$$\nu = K\left(\frac{1}{\tau^2} - \frac{1}{(\tau + \tau_1)^2}\right).$$

This form is exactly equivalent to that previously given, in that it gives every line of the hydrogen spectrum just as the former did, but there is a difference between the two forms in the succession of lines obtained by fixing the value of τ_2 and giving to τ_1 in the first formula, or to τ in the second form, a succession of integral values. In the second form the frequencies obtained evidently fade away for increasing values of τ finally to zero, no matter what value is assigned to τ_2. This is evident because $1/\tau^2$ and $1/(\tau+\tau_2)^2$ tend to equality the larger τ becomes, and their difference tends towards zero. In the original form the frequencies tend to increase to a "head" value as we increase τ_1, letting τ_2 remain fixed.

The author's theory is that an infinite series of frequencies is emitted by the two electrons in one atom as they are returning to their original orbit after being disturbed, and that these frequencies correspond exactly to those in the second equation just given, when τ_2 has some fixed value. On a different occasion τ_2 may have a different value and the electrons go out to a different distance from the nucleus, and give a different series of frequencies on returning. A multitude of atoms will thus give all of the lines observed in the spectrum of hydrogen.

The difference between this theory and the Bohr theory alluded to is that the Bohr theory assumes that one single vibration frequency only is emitted in one operation of an electron, in changing over from a circular orbit of a larger radius to a

circular orbit of a smaller radius. It is impossible under this hypothesis that these frequencies should be due to the orbital motion of the electron, for, the path in changing over from one orbit to another cannot possibly be a simple circular path, and the acceleration cannot, therefore, have a single harmonic frequency as supposed. The Bohr theory offers no explanation whatever of the source of these vibrations, and leaves the matter surrounded in mystery.

By making certain reasonable assumptions concerning the accelerations of the two electrons in the hydrogen atom, the author has worked out possible forms of the orbits described by the two electrons in returning to their original orbit, such that the sum of the accelerations of the two contains only those frequencies of vibration that are contained in a single one of the series of frequencies contained in the second form of frequency equation above given, when τ_2 has one fixed value. The form of path and the rate of motion of the electrons along it gives a complete explanation of the production of these observed frequencies, and no others.

By treating the case by the principle of the conservation of energy, and by the use of the Einstein equation above referred to, it has been possible to arrive at an expression for the energy required to pull the electrons completely away from the nucleus of the hydrogen atom, and from this to obtain numerical values of the so-called ionizing voltages for

hydrogen. For a more complete treatment of this phase of the subject the reader is referred to the book, "The Atom." It can merely be stated here that the ionizing voltages thus obtained for hydrogen are as follows:

τ_2	Ionizing voltage
1	15.496
2	13.806
∞	11.132
3	13.055
4	12.631
5	12.361
6	12.172
7	12.034
8	11.927
9	11.845
10	11.776
....	....

It is considered that these results are in remarkably close agreement with the experimental values of these voltages for hydrogen. It has been observed that ionization sets in at about 11 volts, and a trifle over. This corresponds to the value of $\tau_2 = \infty$ in the above table, and this is the minimum value. Nothing whatever is observed to happen in hydrogen until this voltage is reached. A new type of ionization has also been observed at about 15.8 volts by Davis and Goucher. This is very close to the upper limit, or maximum value in the table corresponding to $\tau_2 = 1$. A distinctive point has also been observed at about 13.6 volts, which corresponds to $\tau_2 = 2$ in the table.

The other intermediate values just above 11.132 volts are so close together that it would not be pos-

sible to distinguish the individual voltages called for corresponding to large values of τ_2, because the experimental data merges these humps into a smooth curve. The ionization is shown by the table to be nearly continuous after 11.132 volts is reached until we come to the small values of τ_2 near the top of the column.

It ought to be stated that the Bohr theory of the atom gives a maximum ionizing voltage of about 13.54 volts, which is not in agreement with the experimental facts. It is considered that the obtaining of numerical values of these voltages in close agreement with observations from the new theory of the atom is strong support for the ideas lying back of the theory.

APPENDIX

The following remarks have been prompted by a sentence in the short paper by A. H. Halloran in the Journal of Electricity for March 15, 1920, in which reference is made to my theory of gravitation. The sentence has particular reference to the old question of the velocity of transmission of the gravitational force, and reads: "Newton's law of **instantaneous** gravitational action, however, is upset by Einstein's deduction that no action can exceed the velocity of light."

No statement as to the velocity of transmission of the gravitational action is to be found in Newton's original statement of the law of gravitation. And, moreover, it was not Einstein who first introduced the conception that all effects propagated through the ether of space take place with the velocity of light. This is the fundamental basis underlying the current form of electromagnetic theory as exemplified by the Lorentz form of this theory.

As soon as we abandon the idea that the gravitational force is an effect peculiar to itself, and not at all connected with electromagnetic phenomena, and adopt instead the form of the gravitational equa-

tion which I have deduced by means of a modified Lorentz theory, modified to make the dimensions of the equation agree with the dimensions of a mechanical force, we have then included gravitation in the realm of electromagnetic phenomena.

This automatically means that none of the effects comprehended within my theory are propagated with a speed exceeding the velocity of light.

It was probably Laplace's celebrated treatment of the motion of a comet under the influence of the sun's attraction that focused the attention of the world on the subject of the velocity of transmission of a gravitational influence, no mention of which is contained in Newton's original statment. So that it now seems advisable to look at the question afresh, that is to say from the standpoint of the author's theory of gravitation.

Laplace made the hypothesis that the gravitational force was due to some unknown influence emanating out in straight lines from the sun, and meeting the comet at each point of its path as it described a hyperbolic orbit around the sun. On this hypothesis he reached the entirely legitimate conclusion that the comet must deviate by an easily observable amount from a true hyperbolic path, if the supposed unknown influence constituting the gravitational force were propagated with a velocity equal to that of light. He showed that the path would deviate less and less as its velocity is increased more and more, but at the same time that the deviation

from the hyperbolic orbit would still remain within observable limits even if the velocity of his supposed gravitational influence exceeded the velocity of light many times.

This gave rise to the notion that this gravitational influence, whatever it was, must travel at a very excessively great speed, say an infinite speed, if we please; for this would really be required to prevent the orbit from deviating theoretically at all from the exact hyperbola.

Let us, therefore, examine more attentively the physical phenomena which form the foundation in electromagnetic theory of the present view of gravitation. An infinite velocity really implies that the effect has already traversed the distance, say from the sun to the comet, and is already there, so to speak, at the point of the orbit in question even before the comet has reached this point of its future path. This is shown to be exactly the state of things when the theory of gravitation described in "The New Physics" is examined in detail.

To fix the ideas, consider the effect of a single electron in space revolving at a uniform rate in a circular orbit, this electron being, so far as we are now concerned, the only one in existence. Let the circle of its orbit be represented by the small circle, e_2, at the right of Fig. 1, in the plane of the paper.

According to the Lorentz electromagnetic theory this revolving electron radiates out its influence in all directions in space, and, assuming now that this

electron has been so revolving for an exceedingly long time, its influence has had sufficient time to travel to the uttermost limits of space, as far as the imagination can travel.

The general form of the mechanical force equation, due to Larmor and Lorentz, which a charge e_2 exerts upon a charge e_1, is

$$\mathbf{F} = e_1 \left(\mathbf{E} + \frac{1}{c} \mathbf{q}_1 \times \mathbf{H} \right),$$

where e_1 represents the electrical charge upon which the force is exerted, and $\mathbf{q}_1$ is the vector velocity of

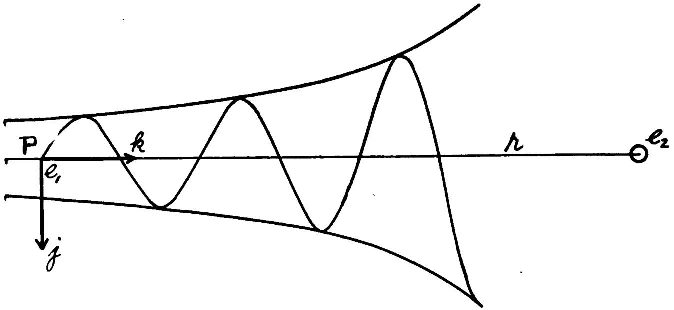

Fig. 1—Diagram to show the effect of a single electron in space revolving at a uniform rate in a circular orbit. The circle of its orbit is represented by the circle e_2 at the right of the figure.

this charge. If there is no second charge, e_1, there is no mechancial force F, the expression reducing to zero. And, again, if there exists a charge, e_1, and its velocity, $\mathbf{q}_1$, is zero, meaning that it is at rest, the second term of the expression vanishes, leaving

$$\mathbf{F} = e_1\mathbf{E}.$$

E and **H** are vector expressions sometimes called the electric and magnetic components of the electromagnetic force, but they only become forces when multiplied by an electrical charge.

Suppose now that a stationary charge, e_1, suddenly appears at some point of space, P, being created if necessary to aid in the mental picture, it will at once and **immediately** experience the force $e_1\mathbf{E}$ corresponding to its location with respect to the revolving electron, e_2, without any delay whatever, for the field **E** is already there, so to speak. This would be true wherever one placed the charge, e_1, for the field of e_2 has already traversed all space, which may be considered to be filled with standing electromagnetic waves.

When the distance is great between e_1 and e_2 it may be shown that the force $e_1\mathbf{E}$ according to the Lorentz theory, assuming the conditions to be as represented in Fig. 1, may be very approximately represented by the equation

$$e_1\mathbf{E} = -\frac{e_1 e_2}{r}\,\frac{\beta_2{}^2}{a_2}\cos\left[\omega_2\left(t - \frac{R}{c}\right) + \theta_2\right]\mathbf{j}.$$

The axes of the reference here, i, j, and k, are centered at the point P, the k-axis coinciding with the line joining centers in the plane of the paper, the j-axis being directed downward in the figure, and the i-axis being upward perpendicular to the paper. The distance from e_2 to e_1 is denoted **R**, and this at great distances is not sensibly different from the

distance from the center of the orbit of e_2 to the fixed e_1, denoted by r. ω_2 is the angular velocity of the electron, e_2, in its orbit, c the velocity of light, a_2 the radius of the orbit of e_2 and β_2 its speed in terms of the velocity of light as unity.

The whole force is evidently a simple harmonic force along the j-axis coinciding in frequency with the revolution of the electron, e_2. At some time in its cycle of variation this force is evidently zero at the point P; but it is not then zero at other points along the line joining centers. When the force is zero the cosine must be zero, and we have

$$\omega_2\left(t - \frac{r_1}{c}\right) + \theta_2 = \frac{\pi}{2},$$

where r_1 now represents the distance to P. As the point P moves toward e_2 we eventually arrive at a new position, r_2, where the force is zero at the same time, and

$$\omega_2\left(t - \frac{r_2}{c}\right) + \theta_2 = \frac{\pi}{2} + 2\pi$$

By subtraction

$$\frac{\omega_2}{c}(r_1 - r_2) = 2\pi$$

$$\text{or}\quad r_1 - r_2 = \frac{2\pi c}{\omega_2} = \frac{2\pi c}{2\pi n} = \frac{c}{n},$$

where n is the frequency of the vibration. This change in the distance is merely the wave-length, λ, since

$$\lambda = \frac{c}{n} = r_1 - r_2.$$

The whole field of force may then be mapped out along the radius r by laying off equal intervals, λ, and by representing the force at intermediate points by a harmonic curve, as in the figure. The amplitude of the harmonic curve, however, must increase as e_2 is approached because of the factor, $1/r$, in the coefficient of the equation. At half the distance the force is doubled, and so on. This curve represents the force at each point at the same instant of time. As time passes, this field would present the appearance of waves traveling outward from e_2 with the velocity of light, although the harmonic force at each point remains fixed and stationary. The case is very analogous to one of the electric advertising signs in which an effect is made to appear to be propagated with considerable velocity, merely by giving a harmonic variation to the light intensity of each fixed lamp.

Similarly, along every radius emanating from the center of the orbit of e_2 there is formed a like picture, and, were these waves visible, they would appear to be streaming out from this center with the velocity of light. But, the velocity with which these waves travel has nothing to do with the so-called velocity of propagation of the gravitational effect. The field is already there, and is definite and fixed at every point of space surrounding e_2 long before the electron e_1 entered the region.

When the second charge, e_1, enters this field, or rather moves about in it, for it could hardly escape always being in some point of the field, the effect that it experiences is due to the existing field already at the point in question, an effect already present, and requiring no time for its transmission from some distant place.

The very fact that the difficulty of Laplace, really injected into the subject because of his concept of the cause of the gravitational effect, has vanished into nothingness by considering the definite physical concepts of the author's theory of gravitation as dependent upon an electromagnetic basis,—this in itself furnishes strong reasons in support of the new theory.

In conclusion, it ought to be pointed out that the forces which have been used above, both in the equation and its graphical representation in Fig. 1, are but one component part of the total force, namely, that part which varies inversely as the distance. There is no component of this inverse distance force that acts along the line of centers, r, so that the author's gravitational force is not represented in this diagram at all.

The magnitude of that part of the force varying as the inverse distance may, however, be made far greater than any component varying as the inverse square or higher powers merely by taking the distance, r, large enough. Assuming that the figure represents large distances only, the force there rep-

resented would not be changed in sufficient degree to be perceptible to the eye by the addition of all other components. This inverse distance component of the force, however, illustrates the matter of the standing waves, that is, waves which do not change their magnitude with time at a given point in space, and takes care of the matter of the velocity of propagation of gravitation with which we are immediately concerned, just as well as if the inverse square component were used alone in the diagram. The reason for illustrating by means of the former instead of the latter, or the real force in question, is because the inverse distance component is capable of simpler mathematical expression, and is adapted to be illustrated by a diagram.

This inverse distance component is by far the greater force, and, while it contributes nothing to the gravitational effect, yet in the author's theory of the atom this is the force that is concerned with the radiation of energy.